Random Field Modeling of Railway Track Irregularities

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(Reviewed by the Highway Division)

Abstract: Track irregularity is considerably important in track-vehicle studies. Recorded data of these irregularities form a basis to develop track standards. This also helps specify track inputs to the vehicles. In this paper random field models for vertical irregularity data obtained from Indian railways has been presented. It is shown that irregularity in the vertical profile can be modeled as stationary Gaussian random field, which can be defined in terms of the power spectral density (PSD) function. Such a model can predict the values of peak amplitudes in a given track length. Further, the absolute vertical profile of the left and right rails belong to the isotropic random field. The implication of this in describing the input matrix of multiaxle vehicles is discussed.

INTRODUCTION

Among the various causes that influence the dynamics of track-train systems, track irregularity is perhaps the most important one. Irregularity is the variation from a long-term average level in the vertical and lateral profiles. These variations in geometry get developed due to the settlement of subgrade, ageing, and ballast loosening. The heterogeneity of materials with depth and nonuniformity of stiffness along the length makes it unnatural to expect a track free of irregularities. Profile irregularities make the vehicles vibrate, which in turn induce stochastic forces on the track. Under these forces the tracks may yield, leading to further deterioration of the track and reduction of comfort. Thus, engineers are left with only the option of maintaining the track irregularities within acceptable limits. The specification of these limits for practical use demands random process (or field) modeling of irregularities. Track irregularities include four random parameters, namely, vertical profile, alignment, cross level, and gauge (Fig. 1). These four random fields are correlated. However, in practice the vertical irregularity is used extensively. (In Fig. 1, y_L = absolute vertical profile of left rail, y_R = absolute vertical profile of right rail, G = gauge, w_L = centerline assignment, and $y_R - y_L/g$ = cross level.)

In the past there have been several studies on quantifying track irregularities. The Office of Research and Experiments (ORE) report *C116* ("Power" 1971) describes power spectral density (PSD) functions of various track profiles obtained from four different railways. Balzer (1978) compared the PSD of railway tracks with the PSD of roads, runways, cow pastures, etc. He emphasized the need for roughness classification based on the PSD. Corbin and Kaufman (1983) analyzed the PSD of track-irregularity data obtained from American railways. They separated the periodic components in the PSD and used it as a diagnostic tool to classify tracks. They also suggested analytical forms for the PSD. Desh (1983) analyzed the vertical unevenness data obtained from Indian railways. This data was measured on 3.6 m and 9.6 m chord base lengths. He compared the PSD functions of this data obtained before and after maintenance to study the rate of track deterioration. He highlighted the effect of moisture content in the subgrade on track deterioration.

There are many similarities between track and road irregularities. Dodds and Robson (1973) and Kamash and Robson (1978) have discussed the PSD description of road roughness. Their important observation was that road surface roughness can be modeled as an isotropic random field. This considerably simplifies the input specifications of vehicles. Honda et al. (1982) determined PSD functions of the road profiles on bridges, using the maximum-entropy method. In a similar work, Marcondes et al. (1991) computed the PSD of highway roughness profiles.

As mentioned, irregularities have to be modeled for track maintenance, vehicle dynamics, and track dynamics. Although the absolute vertical profile (AVP) is needed for vehicle bounce and track dynamics, for maintenance Indian railways generally use irregularity data measured on a sliding chord 3.6 m or 9.6 m long. This data, defined as unevenness, is the AVP data filtered in a particular fashion. For track maintenance, attention is focused on removing those wavelengths in the irregularity that predominately affect the vehicle ride quality. In chord-based measurements, AVP data gets filtered so that these wavelengths are correctly reflected. The maintenance criterion is based on these chord-based measurements. The length of the chord depends on the wavelength of interest, which in turn depends on the vehicle type and velocity. With this in view, this paper presents both types of track data in the vertical direction, namely, unevenness and AVP. With the unevenness data, the criterion for maintenance will be based on the peak amplitude and number of peaks. The sequel demonstrates that both data can be modeled as Gaussian random fields. Further, the AVP data of left and right rails belong to an isotropic random field. The implication of this in describing the track input matrix of multiaxle vehicles is discussed here.

DATA

Indian railways routinely measure the unevenness and AVI of important national routes. Two routes, Delhi – Madras an Delhi – Calcutta, were selected for further analysis. Four kn of unevenness data and 5 km of AVP data was obtained fror each of the two routes. The letters G and R were used t identify samples from routes Delhi – Madras and Delhi – Ca cutta, respectively. The data is recorded for both left an right rails. The unevenness data samples selected are suc that their standard deviations are among the top 5% of th route kilometers. The sampling interval is 0.6 m for the ur evenness data and 0.405 m for AVP data. These data ai

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divided into samples 1 km long. Typical samples of unevenness and AVP are shown in Fig. 2.

STATISTICAL PROPERTIES

The most desirable property in a random process is its Gaussian nature. The simplest way to verify this property is by computing sample moments and sample probability-density functions (PDF). Accordingly, in Tables 1 and 2 the first four moments of the unevenness data and AVP data are presented. The Kolmogorov-Smirnov (K-S) test to verify the Gaussian nature of sample cumulative-frequency distributions is presented in Fig. 3. The K-S test is performed on the pooled data. The skewness of the data is nearly zero and the kurtosis is around three. This indicates that the data may be considered normally distributed. This observation is further supported by the K-S test (Fig. 3). A zero-mean normal or Gaussian random field can be described in terms of its PSD function. Thus, the modeling effort reduces to the computation of PSD functions.

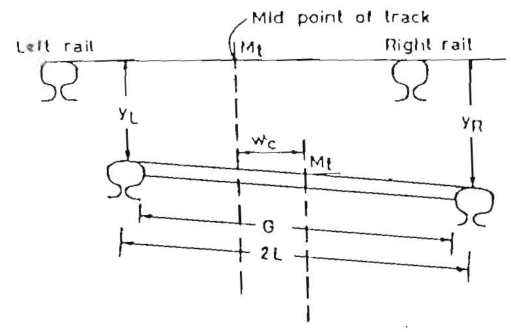

FIG. 1. Track Irregularity Parameters

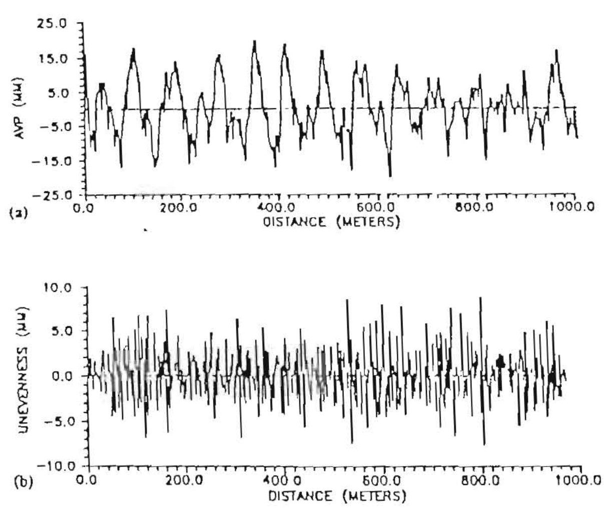

FIG. 2. Typical Data Samples

TABLE 1. Statistics of Unevenness Data

Sample (1)	Mean (mm) (2)	Standard deviation (mm) (3)	Skewness (4)	Kurtosis (5)
GL1	−18.07	2.31	−0.64	3.55
GR1	0.720	2.15	−0.70	4.02
GL2	−17.82	2.39	0.35	4.41
GR2	0.57	2.46	0.70	6.21
GL3	−18.34	2.67	−0.3	3.45
GR3	0.82	2.95	−1.01	6.16
GL4	−18.10	2.50	−0.02	4.25
GR4	0.630	2.67	−0.62	5.96
RL1	8.173	2.44	0.21	3.41
RR1	−16.01	2.71	0.11	3.19
RL2	8.06	2.29	0.65	5.11
RR2	−16.08	2.37	0.5	4.76
RL3	−20.81	2.19	0.49	4.01
RR3	−0.26	2.62	0.06	3.80
RL4	−20.69	1.93	0.41	3.31
RR4	−0.40	2.56	0.25	3.68

TABLE 2. Statistics of AVP Data

Sample (1)	Mean (mm) (2)	Standard deviation (mm) (3)	Skewness (4)	Kurtosis (5)
GL1	0.04	7.82	0.18	2.54
GR1	0.02	6.86	0.15	2.46
GL2	−0.06	6.87	0.31	3.62
GR2	−0.04	5.93	0.13	3.05
GL3	−0.04	7.45	−0.16	3.70
GR3	−0.05	6.00	0.24	2.95
GL4	0.15	7.23	−0.11	3.00
GR4	0.14	5.60	0.00	2.87
GL5	0.02	5.77	−0.13	3.63
GR5	−0.02	5.02	−0.09	3.93
RL1	0.02	7.95	−0.35	3.26
RR1	0.01	8.23	−0.26	3.48
RL2	−0.20	6.66	−0.04	2.65
RR2	−0.20	6.11	−0.05	2.66
RL3	0.20	6.59	−0.02	3.08
RR3	0.19	6.08	−0.08	3.19
RL4	−0.01	4.32	−0.18	3.71
RR4	−0.01	4.48	−0.10	3.39
RL5	0.04	6.27	−0.11	3.32
RR5	0.02	6.22	−0.22	3.55

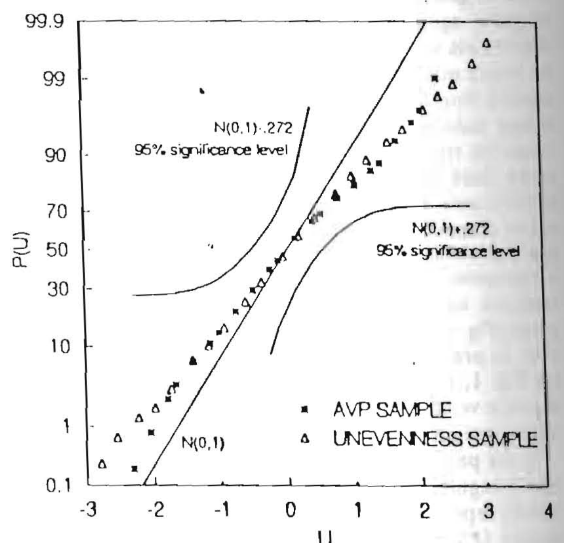

FIG. 3. K-S Test Results

PSD FUNCTIONS

Computation of the PSD function is now routine for stochastic data. There are various algorithms available for the efficient computation of the PSD. For the present data it has been found, by trial and error, that the Blackman-Tukey algorithm (Bendat and Piersol 1971), using the Parzen window gives the most robust estimate of PSD. Such a PSD has been computed for all samples of the two routes. Fig. 4 shows a typical sample PSD for unevenness and AVP data. These

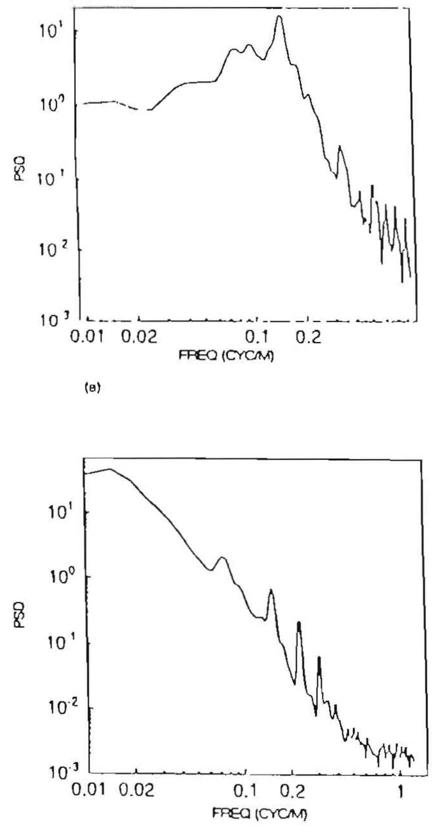

FIG. 4. Sample PSD Functions: (a) Unevenness Sample; (b) AVP Samples, GL1

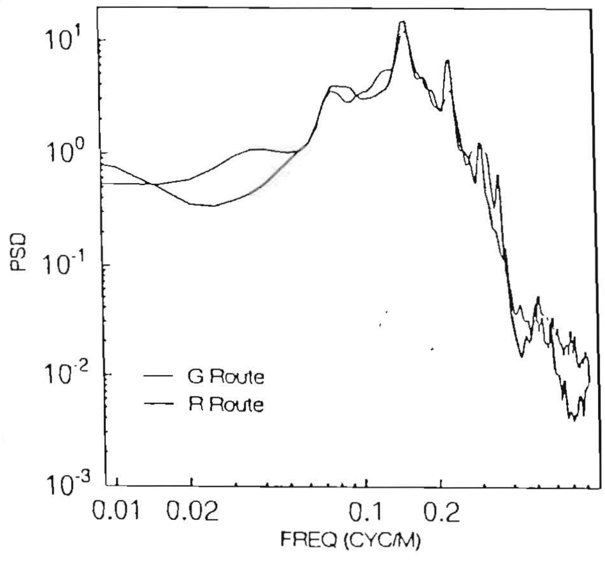

FIG. 5. Standard PSD for Unevenness

are further averaged in the wavelength domain to arrive at the standard route PSD functions shown in Figs. 5 and 6. The data has been normalized by removing the mean and scaled with respect to the standard deviation. Thus, the results in Figs. 5 and 6 refer to the standard PSD shapes only.

For the unevenness data one would be interested in the information on level crossing and peak statistics. The PSD function was used to obtain this information.

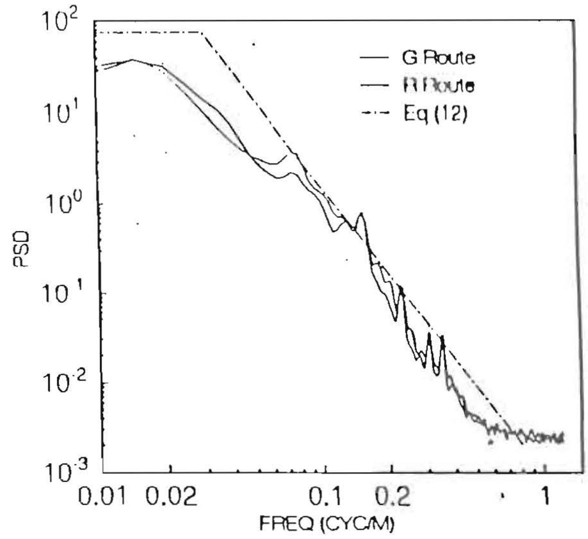

FIG. 6. Standard PSD for AVP

LEVEL CROSSINGS IN UNEVENNESS

Since the data can be taken as a Gaussian random field, the classical level-crossing and peak-statistics theory (Nigam 1983) can be exploited to relate the sample standard deviation to the highest peak value. The PSD of the unevenness process $u(x)$, which is a zero-mean Gaussian stationary random field, is denoted as $S_{uu}(f)$, where f is the spatial frequency in cycles/m. It follows that

$$\sigma_1^2 = \int_0^\infty S_{uu}(f)\,df;\quad \sigma_2^2 = \int_0^\infty f^2 S_{uu}(f)\,df;$$

$$\sigma_4^2 = \int_0^\infty f^4 S_{uu}(f)\,df \tag{1}$$

where σ_2^2 and σ_4^2 = variance of the first and second derivative process of $u(x)$, respectively. The average number of zeros in a length L will be

$$N_0(L) = (\sigma_2/\sigma_1)L \tag{2}$$

Similarly, the average number of peaks in a length L will be

$$N_p(L) = (\sigma_4/\sigma_2)L \tag{3}$$

Further, the probability of a peak being greater than a level $\alpha = (a/\sigma_1)$ at any point is

$$P(\alpha) = 0.5\{1 - \mathrm{erf}[\alpha/(\sqrt{2}\beta)]\} + 0.5(1 - \beta^2)^{0.5}\exp(-0.5\alpha^2) \cdot \{1 + \mathrm{erf}[\alpha(1 - \beta^2)^{0.5}/(\sqrt{2}\beta)]\} \tag{4}$$

where β = bandwidth parameter

$$\beta = [1 - \sigma_2^4/(\sigma_1^2\sigma_4^2)]^{0.5} \tag{5}$$

The values σ_2, σ_4, and β for all the unevenness data samples are presented in Table 3. The comparison between observed and estimated average number of level crossing and peaks is presented in Table 4 for four samples.

ORDER STATISTICS OF PEAKS

With reference to Fig. 7, let there be N number of peaks in a particular track stretch of length L. These peaks are denoted as $a_1, a_2, \ldots, a_N$ and are arranged in descending order as

$$a_1 > a_2 > a_3 > \cdots > a_N \tag{6}$$

TABLE 3. σ_2, σ_4, and β Values for Unevenness Data

Sample (1)	σ_2^2 (2)	σ_4^2 (mm^{-2}) (3)	β (4)
GL1	0.15	0.0065	0.558
GR1	0.14	0.0063	0.549
GL2	0.14	0.0055	0.596
GR2	0.16	0.0067	0.571
GL3	0.24	0.0131	0.624
GR3	0.3	0.0170	0.625
GL4	0.18	0.0086	0.631
GR4	0.24	0.0125	0.584
RL1	0.12	0.0043	0.641
RR1	0.16	0.0062	0.619
RL2	0.13	0.0059	0.647
RR2	0.15	0.0070	0.655
RL3	0.16	0.0082	0.579
RR3	0.3	0.0201	0.589
RL4	0.13	0.0079	0.604
RR4	0.287	0.0189	0.575

TABLE 4. Crossing Statistics for Unevenness

Sample (1)	Upward zeros observed (2)	Upward zeros estimated (3)	Number of peaks observed (4)	Number of peaks estimated (5)
GR1	158	171	182	205
GR2	149	161	169	197
RL1	133	140	156	183
RL4	174	187	204	235

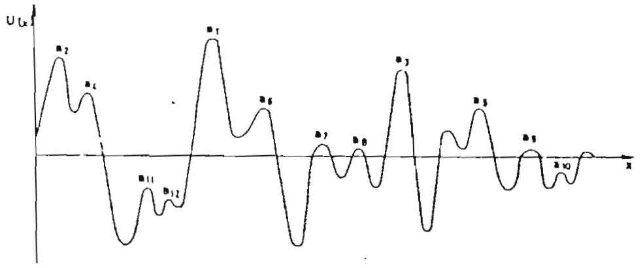

FIG. 7. Peak Amplitudes of Various Order

The probability that the jth-order peak will exceed the level $\alpha = (a/\sigma_1)$ is

$$F_j(\alpha) = \text{Prob}(a_j > \alpha)$$

$$= \text{Prob (at least } j \text{ number of } a_i \text{ are greater than } \alpha) \quad (7)$$

Now, assuming the peaks are statistically independent, (7) is equivalent to

$$F_j(\alpha) = P\text{ (only } j \text{ peaks} > \alpha) + P\text{ (only } j + 1 \text{ peaks} > \alpha)$$

$$+ P\text{ (only } j + 2 \text{ peaks} > \alpha)$$

$$+ \cdots + P\text{ (only } N \text{ peaks} > \alpha)$$

$$= \sum_{i=j}^{N} P\text{ (only } i \text{ out of } N \text{ peaks} > \alpha) \quad (8)$$

From probability theory

$$P\text{ (only } i \text{ out of } N \text{ peaks} > \alpha) = {}_NC_iP^i(\alpha)[1 - P(\alpha)]^{N-i} \quad (9)$$

$$\text{where } {}_NC_i = (N!)/[i!(N - i)!] \quad (10)$$

and $P(\alpha)$ is given by (4). Thus, finally

$$F_j(\alpha) = \sum_{i=j}^{N} {}_NC_iP^i(\alpha)[1 - P(\alpha)]^{N-i} \quad (11)$$

is the probability that the jth-order peak among the N number of peaks in a track length L will exceed a given level $\alpha = a/\sigma_1$. Thus, if a low probability of exceedance, say 1%, is prescribed, $F_j(\alpha) = 0.01$, (11) can be solved to find the corresponding value of α, at any specified order j. In Table 5, for the unevenness sample observed peak, values are compared with the predicted values at the 1% exceedance level. PSD samples are used to evaluate various quantities in (1).

HIGHEST PEAK AMPLITUDE

The comparisons shown in Table 5 are very favorable and support the proposed Gaussian model. This indicates that from the knowledge of PSD function of a stretch of track, information on peak amplitudes in that stretch can be obtained. However, for routine practical application one would like to measure the simplest statistics and relate it to the peak amplitude of the unevenness in a given length, e.g., 200 m of the track. It appears that standard deviation can be reliably and quickly measured in the field. Hence, a maintenance engineer would like to base the decision on this quantity, alone. This observation clarifies that if standard PSD shapes are available, the actual PSD for a given stretch can be obtained by multiplying standard PSD shapes by the sample standard deviation. With the help of this PSD one can estimate the highest peak amplitude and decisions on the track quality can be made. In Figs. 8 and 9, the predicted average value of the highest peak is plotted as a function of the stan-

TABLE 5. Peak Amplitudes at 1% Level of Exceedance

	Peak Order				
	1 (mm)	2 (mm)	3 (mm)	4 (mm)	5 (mm)
	(*a*) Sample GR1				
Observed	6.12	5.53	4.94	4.55	4.50
Estimated	9.51	8.08	7.44	7.03	6.73
	(*b*) Sample GR2				
Observed	8.80	8.60	7.82	6.84	6.26
Estimated	10.72	9.11	8.37	7.91	7.57
	(*c*) Sample RL1				
Observed	8.42	7.84	7.64	6.86	6.66
Estimated	10.62	8.99	8.25	7.78	7.44
	(*d*) Sample RL4				
Observed	7.22	6.80	6.05	5.90	5.85
Estimated	8.45	7.20	6.64	6.28	6.00

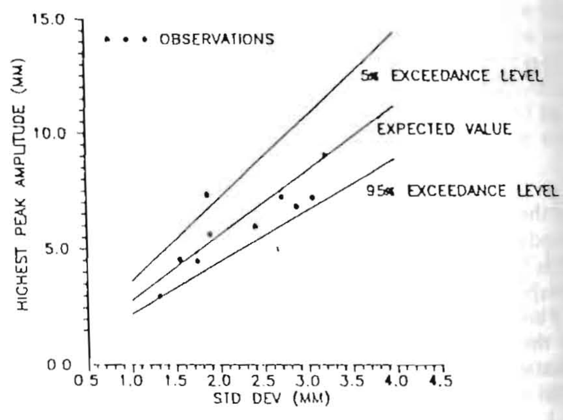

FIG. 8. Highest Peak Amplitude (G Route)

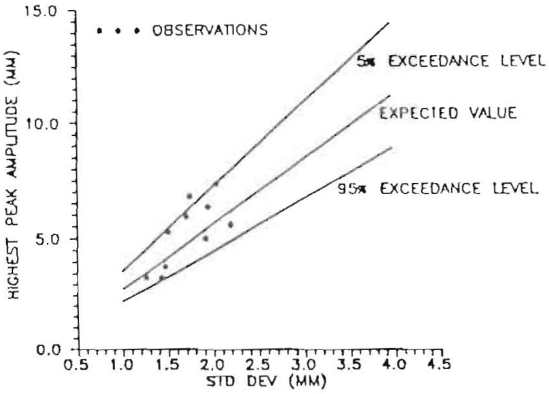

FIG. 9. Highest Peak Amplitude (R Route)

dard deviation, for a track length of 200 m. The 0.05 and 0.95 percentile lines are also drawn to show the 90% confidence band in which the prediction of the highest peak should lie. Also shown are actual, observed highest peak amplitudes of several sample track stretches not included in the original database of Table 1. Once the PSD of a route is standardized, the highest peak amplitude can be estimated for engineering purposes, from charts similar to Figs. 8 and 9, using only the standard deviation value.

PSD OF AVP

The unevenness data analyzed earlier does not contain information at all wavelengths. AVP data containing information at all the wavelengths is useful in track and vehicle dynamics. As explained earlier, PSD samples are ensemble-averaged to arrive at standard AVP and PSD functions. These are shown in Fig. 6 for the G and R routes; both PSDs are similar. There is a concentration of energy at a wavelength of about 14 m, which corresponds with the distance between successive welds on the rails. If these peaks are overlooked and smoothened, the PSD can be described through a power law of the type

$$S(f) = af_1^{-b},\ 0 < f < f_1 \quad (12a)$$

$$S(f) = af^{-b},\ f_1 < f < f_2 \quad (12b)$$

$$S(f) = 0,\ f > f_2 \quad (12c)$$

Here, f_1 and f_2 = suitable cutoff frequencies; and a, b = constants. In Fig. 6, this function has also been plotted for a = 0.001, b = 3.2. Values of f_1 and f_2 are taken to be 0.03 cycles/m and 0.7 cycles/m. These values are for standard PSD shape functions of the two routes. For the actual PSD of any stretch, (12) will have to be multiplied by the variance of that stretch. The standard deviation of AVP is 6 mm to 8 mm for various samples. At greater computational effort, one can fit other types of functions. One may also use the standardized PSD of AVP (Fig. 6) directly instead of fitting a mathematical expression.

ISOTROPY OF AVP FIELD

For any railway track, the AVP data usually corresponds to the left and right rails. In earlier PSD analysis, distinction was not made between these two sets of data. From the track engineering point of view this may not be significant, but for vehicle dynamic studies it is necessary to specify the inputs at all wheel contact points. Thus, for a simple two-axle vehicle (Fig. 10) on a track of gauge G, the input PSD matrix will be S_{ij}, $(i, j = 1, 2, 3, 4)$. In general this matrix will be complex

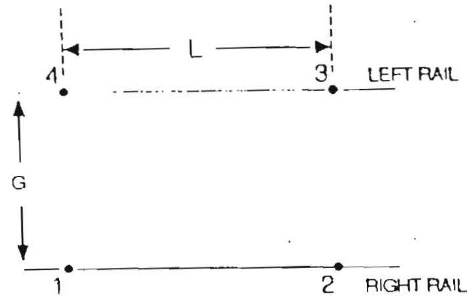

FIG. 10. Input for Two-Axle Vehicle

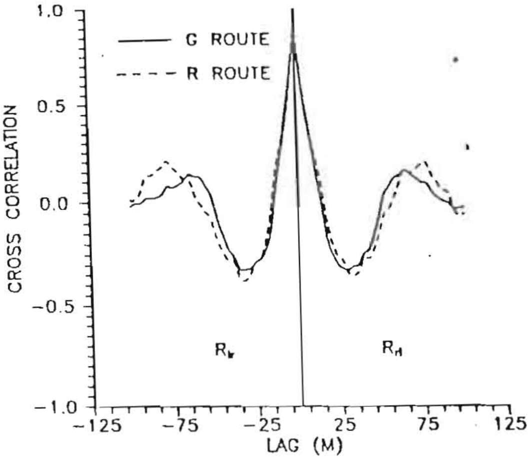

FIG. 11. Cross-Correleation Functions

because the cross-PSD functions of the left and right rails are involved. Fortunately, for the data considered here, one can demonstrate the existence of isotropy (Vanmarcke 1983). This greatly simplifies the vehicle input PSD matrix. In Fig. 11 the sample cross-correlation functions of the left and right AVP data are averaged and plotted, for both the G and R routes. The interesting feature is the apparent symmetry of the figure. Although individual track stretches show departure from this symmetry, for practical purposes, it is proposed that the cross-correlation function is such that

$$R_{rl}(s) = R_{lr}(-s) \quad (13)$$

where s = spatial lag (in meters). For this case, the cross-PSD terms in S_{ij} will be real. Further, for such a random field, the cross-correlation function between a pair of points depends only on the distance between the two points and not on their relative orientation. By using this property, one can easily write the cross-correlation functions, for excitations at various points. For example, with reference to Fig. 10

$$R_{14}(\tau) = R_{41}(\tau) = R_{23}(\tau) = R_{32}(\tau) = R_{11}(\sqrt{\tau^2 + G^2}) \quad (14a)$$

$$R_{13}(\tau) = R_{31}(\tau) = R_{24}(\tau) = R_{42}(\tau)$$
$$= R_{11}(\sqrt{\tau^2 + G^2 + L^2}) \quad (14b)$$

Further, for a vehicle moving at a uniform velocity V, for points on the same rail, the input cross-correlation terms are

$$R_{12}(\tau) = R_{43}(\tau) = R_{11}(\tau + L/V) \quad (15a)$$

$$R_{21}(\tau) = R_{34}(\tau) = R_{11}(\tau - L/V) \quad (15b)$$

From (14) and (15), all cross-correlation terms can be obtained based on knowledge of the PSD or correlations function of the AVP. Thus, the track input matrix can be fully expressed in terms of the PSD function of AVP. If one uses the functional form proposed in (12), the expression for $R_{11}(\tau)$ cannot be written in closed form. However, if other functional forms such as exponential functions are used, one can express all the cross PSDs in analytical form.

COMMENT

Random process modeling has found widespread acceptance in areas of modern engineering applications such as earthquake, wind, and offshore engineering. Railway engineers are also using statistical concepts extensively to understand track and vehicle behavior. The concept of PSD to describe the track and inputs to vehicles has been in vogue for quite some time. In this study, the theory of random fields has been used to obtain level-crossing statistics and peak distribution for unevenness data. It is shown that a stationary Gaussian random model can very well model the peak amplitudes. Further, it is shown that for engineering purposes, from the knowledge of standard deviation over a given track stretch, one can predict the value of the highest peak amplitude. The sample values of skewness and kurtosis indicate departures from the Gaussian nature. Thus, the present Gaussian model is only an approximation, particularly for the unevenness data. It is natural to question how this can be improved to include nonzero skewness and large deviations of kurtosis from three. Random field/process modeling also has to include second-order moment properties, which are currently limited to autocorrelation or the PSD function. A non-Gaussian model, which has a Gaussian process as its first term and can exactly simulate the first four moments, was recently proposed by Iyengar and Jaiswal (1993). An advantage of this model is similar to that of a Gaussian process; it can also be completely described in terms of the PSD function. The application of such models to track data would significantly improve the estimation of peak amplitudes.

An interesting property of the data analyzed here is the isotropy of the AVP. The cross-correlation function between the left and right tracks exhibits symmetry with respect to the lag distance, as shown in Fig. 11. This property can be exploited to simplify the track input specification of vehicles.

In this paper the random field theory has been used to model irregularity in the vertical track profile. A similar analysis needs to made for irregularities in the lateral direction. In the field of transportation engineering there are parameters such as forces on vehicles, material properties, and damage along pavements/tracks, which are stochastic in nature.

CONCLUSIONS

Based on the analysis of two Indian railway tracks, the following conclusions can be made.

Both track unevenness and AVP data can be modeled as stationary Gaussian fields.

Peak values in the unevenness can be estimated from standard deviation.

The vertical irregularity (AVP) corresponding with the left and right rails belongs to an isotropic random field.

The vertical excitation PSD matrix of multiaxle vehicles moving on an isotropic random field can be specified in terms of only one track PSD function.

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APPENDIX I. REFERENCES

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APPENDIX. II NOTATION

The following symbols are used in this paper:

a, b = constants in (12);
$a_1, a_2, \ldots, a_N$ = peak amplitudes;
$F_j(\alpha)$ = probability of jth-order peak > α;
f = frequency in cycles/m;
i, j = indexing parameters;
L = length of track stretch;
N = total number of peaks;
$N_p(L)$ = average number of peaks;
$N_0(L)$ = average number of zero crossings;
$P(\alpha)$ = probability-distribution function of peaks;
S = spatial lag;
$u(x)$ = unevenness process;
α = nondimensional level;
β = band width parameter;
σ_1 = standard deviation of $u(x)$;
σ_2 = standard deviation of first derivative process;
σ_3 = standard deviation of second derivative process; and
τ = time lag.