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NUMERICAL INVESTIGATION OF THE EFFECT OF COMBINED LOADING ON THE LATERAL RESPONSE OF PILES IN SAND

S. Karthigeyan

Geotechnical Engg. Division, Central Building Research Institute, Roorkee

ABSTRACT

The paper presents some results from three-dimensional finite element analyses of piles under combined vertical and lateral loading. The analyses were performed to investigate the influence of combined loading on the lateral response of piles in homogeneous sandy soils. The results have shown that the influence of combined loading on the lateral response of piles is to significantly increase the lateral capacity in sandy soils. In general, it was found that the effect of combined loading is significant even for long piles, which are as long as 30 times the pile width. The design bending moments developed in the pile section were also found to be dependent on the combination of vertical and lateral loads.

INTRODUCTION

Pile foundations are frequently used to support various structures built on loose/soft soils, where shallow foundations would undergo excessive settlements or shear failure. These piles are used to support vertical loads, lateral loads and combination of vertical and lateral loads. However, in view of the complexity involved in analyzing the piles under combined loading, the current practice is to analyze the piles independently for vertical loads to determine their bearing capacity and settlement and for the lateral load to determine their flexural behavior. Procedures are available [7] for analyzing the piles separately under pure vertical loads and pure lateral loads. This approach holds good as long as the magnitudes of the lateral loads are nominal, say less than 5 % of the vertical loads. However, in situations where the lateral loads are significantly high; the interaction effect due to the combined vertical and lateral loads can become critical, which calls for a systematic analysis.

Several investigators have attempted to study the behaviour of single pile and pile groups under pure lateral loads [12, 16]. Besides, with the advent of latest generation computers, it is now possible to investigate the effects due to non-linearity and elasto-plasticity of soil medium, asymmetric loading on piles etc. using 3-dimensional finite element analysis [14, 4, 20]. However, there is hardly any concerted effort to study the influence of combined loading on the lateral response of piles and the literature on combination of vertical and lateral loads is scanty. The limited information on this aspect based on the analytical investigations [5, 17, 6] reveals that for a given lateral load, the lateral deflection increases with the combination of vertical loads. However, experimental [15, 18, 8] and field investigations [13, 2, 21] suggest a decrease in lateral deflection with the combination of vertical loads. Anagnostopoulos and Georgiadis [1] attempted to explain this phenomenon through an experimental model supported by 2-dimensional finite element analysis and reported that the modified status of soil stresses and local plastic volume changes in the soil continuum under combined loads cannot be accounted for in general by the conventional subgrade reaction, elastic half space and other 2-d approaches and thus a nonlinear 3-D finite element analysis would be the most appropriate approach. Trochanis et al. [19] attempted to study the axial response of piles with the combination of lateral loads through 3-d finite element analysis. However, since the piles are not often adequately designed to resist lateral loads, Karthigeyan et al. [10, 11] attempted to investigate the lateral response of piles with the combination of vertical loads through 3-d finite element analysis. In the similar line, the paper presents the numerical results to investigate the influence of combined loading on the lateral response of piles with respect to various design parameters in homogeneous sandy soils.

DETAILS OF NUMERICAL MODEL

All finite element analyses in this investigation were performed using the 3-dimensional finite element program GEOFEM3D. The program is supported by a pre-processor to develop 3-dimensional meshes consisting of bar and beam type prismatic elements, 8-node or 20-node isoparametric brick elements, 8 or 16-node zero thickness type interface elements as well as a post-processing tool that is capable of plotting the original mesh, deformed mesh, displacement vectors, extracting nodal displacements and element stresses along a line/selected plane, etc.

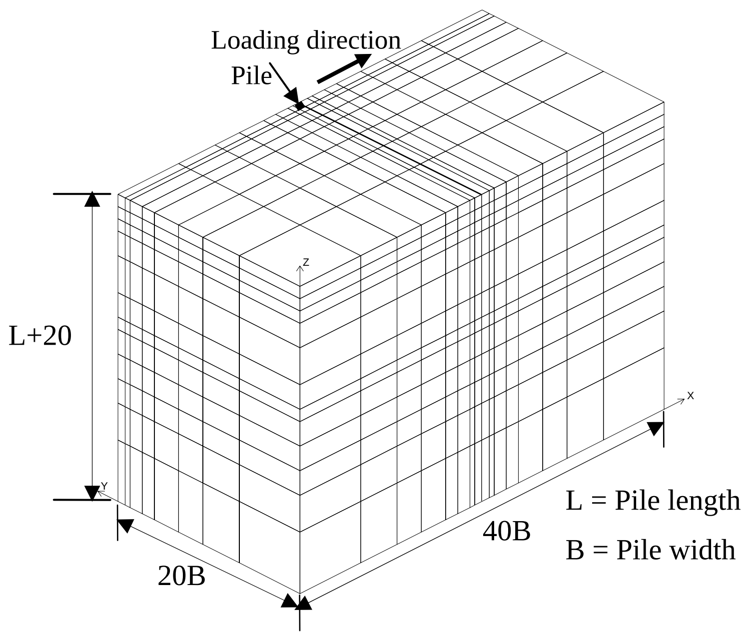


Figure 1. Typical 3-D finite element mesh used in the analysis

Finite Element Mesh Details

Figure 1 shows a schematic 3-d finite element mesh for analysis of pile-soil interaction under combined loading. Based on the symmetry, only half the pile section in the direction of lateral load is analysed (in Figure 1, lateral load is applied along x-axis). 20-node brick elements are used to mesh the pile and the soil continuum. The interface between the pile and the soil has been modelled using 16-node joint elements of zero thickness. All numerical computations of isoparametric elements were performed using reduced numerical integration. The mesh dimensions, number of nodes and elements in the mesh were decided after performing a number of initial trial analyses with several meshes of increasing refinement until the results (displacements and stresses around the pile) did not change significantly with further refinement. The aspect ratios of elements used in the mesh ranged from 0.5 near the pile surface to nearly 5 at the boundaries. The distances to lateral rigid boundaries in the finite element analyses are shown in Figure 1. All the nodes on the lateral boundaries were restrained from moving in the normal direction to the respective surfaces representing rigid, smooth lateral boundaries. All the nodes on the bottom surface were restrained in all the three directions representing rough, rigid bottom surface. Typically, the meshes consisted of approximately 8300 nodes, 1440 20-node brick elements and 52 16-node interface elements.

Pile-Soil Details

In the analysis, the pile was treated as a linear elastic material and the Drucker-Prager constitutive model with non-associated flow rule [19, 20] were used to predict the stress-strain behaviour of soils. This model has been preferred in view of their adaptability to define the failure criterion with the use of simple physical properties such as cohesion (c) and friction angle (ϕ). The failure criterion for the Drucker-Prager model used for analysis has the form $F = \alpha\, J_1 + \sqrt{J_{2d}} - k$, in which J_1 is the first invariant of the stress tensor, J_{2d} is the second invariant of the deviatoric stress tensor and α, k are material constants expressed in terms of the well-known shear strength parameters of soil viz. c and ϕ.

Analysis Scheme

The finite element analyses were performed in two stages. In the first stage, the in situ stresses were initialized in the soil by performing a dummy analysis using a modified Poison's ratio (μ') expressed in terms of the at rest earth pressure coefficient K_o as $\mu'=K_o/(1+K_o)$. The value of K_o it self was obtained as $K_o = 1-\sin\phi$. During this stage of analysis, both the pile and the soil elements were assigned the same material properties corresponding to the soil (Young's modulus, Poisson's ratio and unit weight) so as not to generate any extraneous shear stresses. At the end of this stage of analysis, all the deformations and strains are set to zero to define the datum level for further analysis. During the second stage of analysis, the actual properties of the soil and the pile elements were assigned to them. The set of pile-soil properties considered in the analyses are summarized in Table 1.

Table 1. Properties of pile and soil

Pile details	Sandy soil details
Size : 1200 × 1200 mm square	Undrained cohesion c_u = 0 kPa
Length: 10 m	Friction angle (ϕ) = 36°
Type of pile: Concrete	Dilation angle (ψ) = 12°
Grade of concrete: M25	Young's modulus ($E_{s)}$ = 50 MPa
Young's modulus E_p: 25,000 MPa	Poisson's ratio (μ_s) = 0.30
Unit weight (γ_p): 24 kN/m^3	Unit weight (γ_s) = 20 kN/m^3
Poisson's ratio μ_p: 0.15	Earth pressure (K_o) = 0.41

The shear strength of the interfaces was defined with zero cohesive strength and 2/3rd the friction angle. The interface strength values depend very much on the type of pile material (wood, steel or concrete) and method of installation (driven or bored). The interface strength properties selected for the analyses fall within the range of properties recommended for estimating the skin friction capacity of piles, Bowles [3]. The normal and shear stiffness of the interface elements were initially set to 10^6 kN/m^2/m. These values were decided after performing several analyses with different interface stiffness values. After the shear failure of the interface, the shear stiffness is set to 0.1% of the initial value to permit the relative slip between different materials. The normal stiffness of the interface is set to 0.1% of the initial value when tensile normal stresses develop in order to permit the separation between the pile and the soil.

The external loads were applied in small increments in several load steps with several iterations to satisfy the equilibrium of the system. The iterations were continued at each load step until the norms of out-of-balance force and the incremental displacements were less than 0.5% or until 50 iterations are completed. The analyses were performed using partial Newton-Raphson scheme by updating the stiffness matrix only at the first iteration of each load step.

Validation of the Numerical Model

The validity of the numerical model employed in the program was verified by predicting the pile load test data from the published case of a short rigid pile under combined vertical and lateral loads. The details of this case are presented in the following sub-sections.

Case study [9]

The length and diameter of the concrete test pile considered by Karasev et al. [9] were 3m and 600 mm respectively. The pile was installed in a soil strata consisting of very stiff sandy loam in the top 6m and underlain by sandy clay of lower stiffness of more than 7 m thickness. The shear strength parameters of the topsoil layer were reported as c = 18 kPa and ϕ = 18° and that of the bottom layer as c = 24 kPa and ϕ = 14°. The Young's modulus and the Poisson's ratio of the top soil were taken as 25,000 kPa and 0.35 respectively based on empirical correlations, Bowles (1988). The soil in the bottom layer was assumed to have Young's modulus of 20,000 kPa and Poisson's ratio of 0.40. The dilation angles of the soil in both layers were assumed to be 0° . The field tests were conducted by first loading the pile in the vertical direction and then the horizontal loads were applied while the vertical load was kept constant.

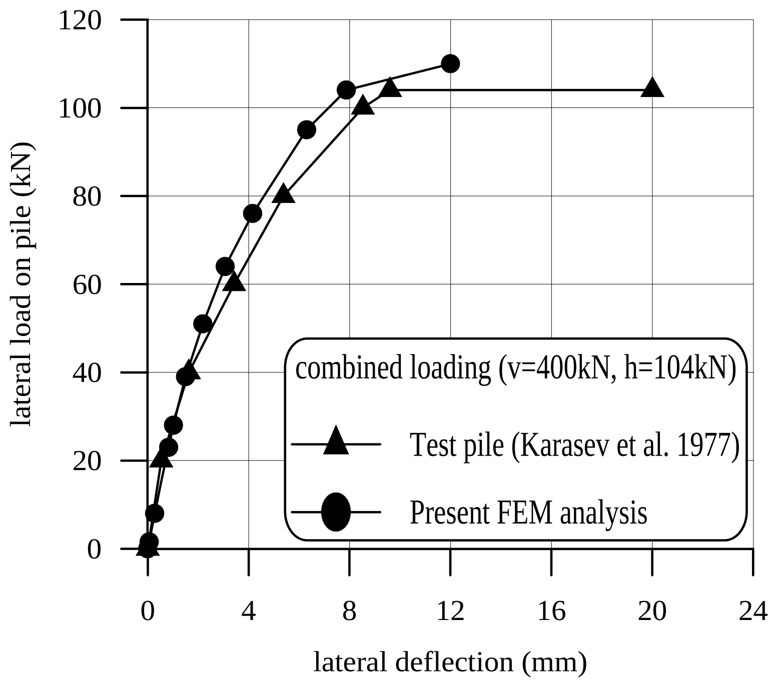


Figure 2. Comparison of finite element results with field test data

The same sequence of load application was followed in the current finite element analysis. The behaviour of the soil was modelled using the Drucker-Prager constitutive model. The pile and the soil were modelled using 20 node brick elements. Using symmetry, only half of the pile was considered in the analysis. The comparison between the finite element predicted and the reported data is shown in Figure 2. The finite element analysis diverged at a load increment of 110 kN which can be considered as numerical collapse load. The numerical analysis was stopped at this stage. The difference in the deformations is because the soil stiffness properties were derived approximately through empirical correlations based on the soil description. Nevertheless the comparison can be considered as good for all practical design purposes.

The finite element prediction in this case matched reasonably well with the test data. Hence, it could be concluded that the numerical scheme adopted in the present investigation is capable of modelling the pile-soil interaction under combined vertical and lateral loads.

PARAMETRIC STUDIES

A series of 3-d finite element analyses were performed on single free-headed pile in homogeneous sandy soils. The dimensions of the pile and the soil properties considered in these analyses are reported in Table 1. The response of the piles under pure lateral load was analyzed initially. For studying the response of piles under combined vertical and lateral loads, the influence of vertical load equal to $0.2V_{ult}$, $0.4V_{ult}$, $0.6V_{ult}$ and $0.8V_{ult}$ has been considered (where V_{ult} is the ultimate vertical load capacity, evaluated a priori by a separate numerical analysis). The ultimate vertical load (V_{ult}) capacity was estimated as 4000 kN (corresponding to the point with maximum curvature on the vertical load-settlement response) by through analysis of a single pile subjected to pure vertical load.

The combined vertical and lateral loads are applied in two stages. In the first stage, vertical loads were applied and then in the second stage, lateral loads were applied while the vertical load was kept constant. This type of loading is similar to that in field situations like the pile jetties, high rise buildings, transmission line towers, overhead water tanks, etc. Here, the piles are first subjected to vertical loading from the weight of the deck or super structure, etc. The lateral loading may be due to wind, wave loading, ship impact, etc. while the piles are subjected to vertical loads. The analysis in the lateral direction was performed using displacement control (rather than load control) so as to be able to evaluate the lateral loads developed at various lateral displacement levels as a percentage of the size of the pile. The reaction forces developed at the nodes were used to calculate the lateral load corresponding to the applied lateral displacements. The numerical results under pure lateral loads and combined vertical and lateral loads on piles are presented and discussed in the following sections. A few analyses were also performed with different slenderness ratios (L/B) of piles to study its influence on piles with combined loading.

RESULTS AND DISCUSSIONS

Figure 3 shows the influence of combined loading on the lateral response of piles in sandy soils. From the data presented, it is noted that there is a considerable increase in the lateral load capacity under increased vertical load levels. The loads shown in the figure correspond to the symmetric half of the pile section. It can be noted that there is a significant increase in the lateral load capacities, in the order of 7 % to 40 %, at deflection levels of 0.05B and of the order of 6 % to 33 % at deflection levels of 0.1B.

The reason for increase in the lateral capacity under the action of vertical load has been examined through the contours of lateral stresses (σ_{xx}) in the vertical plane for piles under pure lateral load and with the presence of a vertical load. Figure 4a and 4b shows typical lateral stress contours in xz plane of piles under pure lateral load and with the presence of a vertical load of $0.8V_{ult}$. It can be seen from the figure that the lateral soil stresses around the

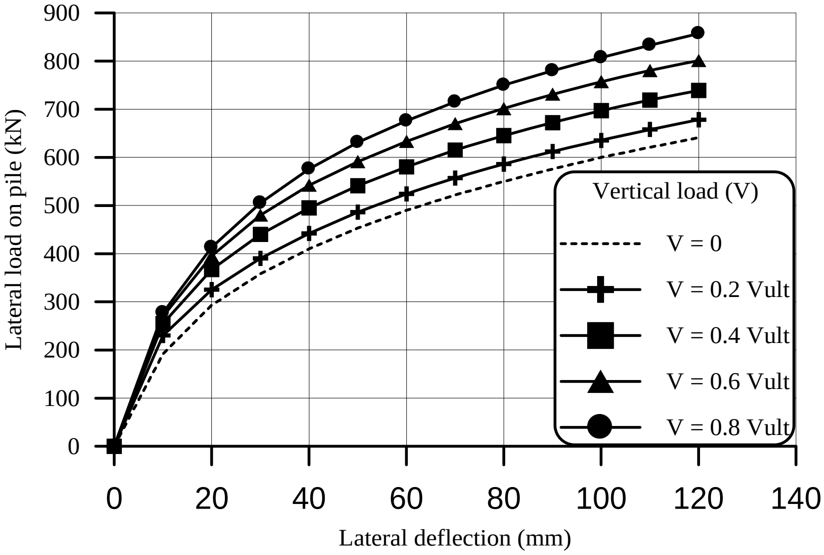
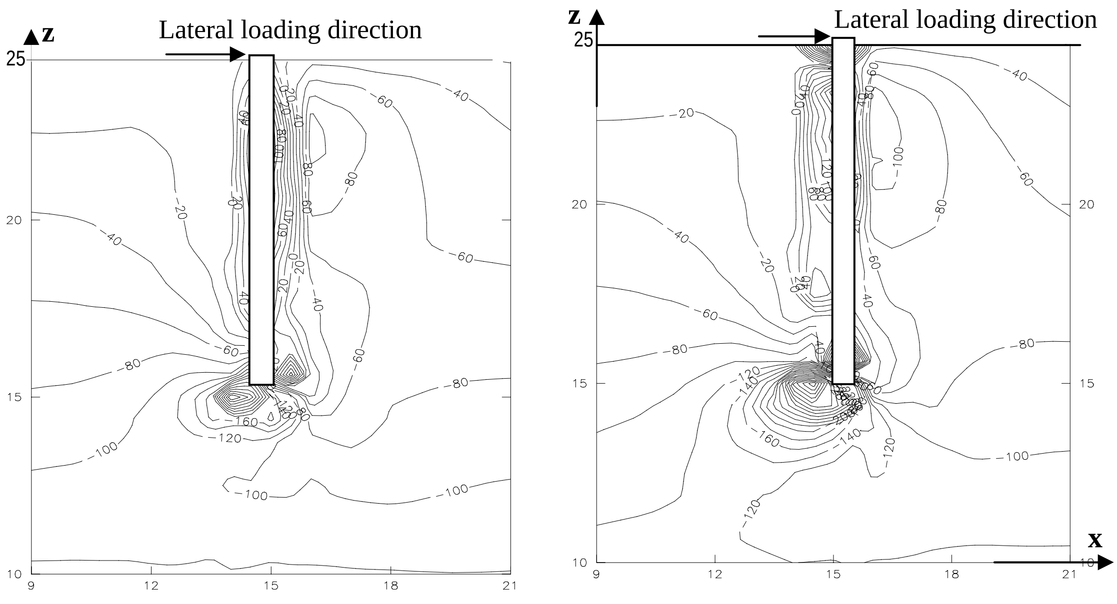


Figure 3. Lateral load – deflection curves of piles under combined loading

piles and along with the depth are substantially higher in the presence of vertical load (Fig.4b) as compared to the pure lateral load (Fig.4a). This increase can be directly attributed to the increase in confining stress at different depths caused by the action of vertical load on the pile. This increased confining stress allows for the development of larger lateral and shear stresses along the pile's frictional face as a result of the increased shear strength of the soil.

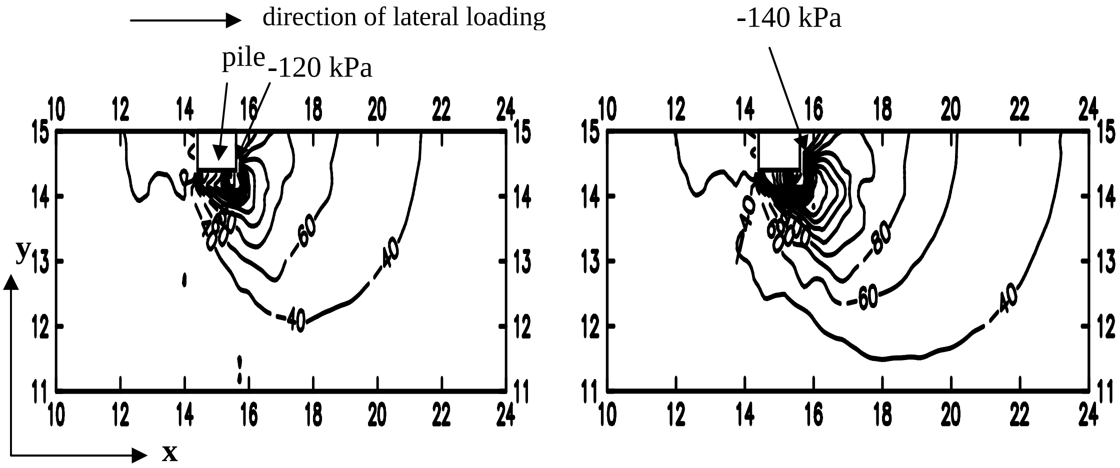


(a) σ_{xx} under pure lateral load **(b) σ_{xx} with vertical load of $0.8V_{ult}$**

Figure 4. Lateral stress (σ_{xx} in kPa) contours in xz-plane of free head piles at a deflection of 0.1B

The increase in lateral soil stresses is further examined through the contours of lateral stresses (σ_{xx}) around the pile under pure lateral load and with the presence of a vertical load of $0.8V_{ult}$

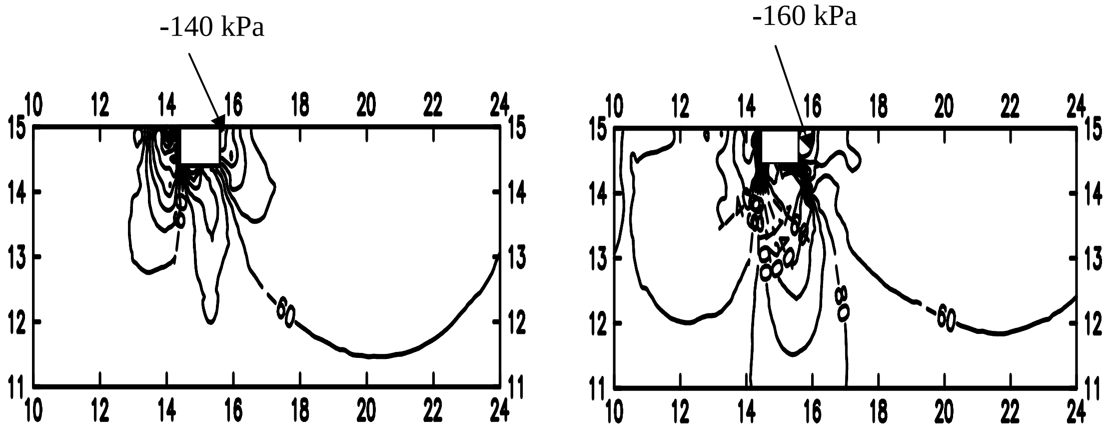
in Figure 5a and 5b. These contours are plotted for a lateral deflection equal to 0.1B and at a depth of 2.8 m from the ground surface (the depth where maximum lateral soil stresses occur) in xy plane.



(a) σ_{xx} under pure lateral load **(b) σ_{xx} with vertical load of $0.8V_{ult}$**

Figure 5. Lateral stress contours in xy-plane at 2.8 m depth from ground surface

Similarly, the increase in shear stresses (σ_{xy}) over the frictional face of the pile is also examined through the contours of shear stresses around the pile under pure lateral load and with the presence of a vertical load of $0.8V_{ult}$ in Figure 6a and 6b. These contours are plotted at a lateral deflection of 0.1B and at a depth of 2.8 m from the ground surface (i.e. the depth at which maximum shear stress occurs). It is clear that the lateral soil stress and the mobilized shear stresses of soil around the pile are higher in the presence of vertical load as compared to the pure lateral load case.



(a) σ_{xy} under pure lateral load **(b) σ_{xy} with vertical load of $0.8V_{ult}$**

Figure 6. Shear stress contours (σ_{xy}) in xy-plane at 2.8 m depth from ground surface

Influence of Pile Slenderness Ratio (L/B)

The influence of pile slenderness ratio (L/B) under combined vertical and lateral loading was studied by performing 3-dimensional finite element analysis of 600 × 600 mm size square pile with different lengths of 5, 10, 15 and 20 m. Similarly, 3-dimensional finite element analyses were also performed by keeping the pile length constant at 10 m and varying the pile widths to 300, 400, 600, 900 and 1200 mm. All these analyses were performed in the same manner as described earlier. Based on the numerical results obtained, the **P**ercentage **V**ariation in lateral **C**apacity (PVC) with respect to different levels of vertical loads is calculated for various L/B ratios with respect to various pile length (L) and widths (B) considered in the analysis. The PVC is defined as follows in terms of the **L**ateral load **C**apacity **W**ith **V**ertical load (LCWV) and the **L**ateral load **C**apacity under **P**ure **L**ateral loading (LCPL),

$$PVC = \frac{LCWV - LCPL}{LCPL} \times 100\% \qquad (1)$$

The results have shown that the influence of combined loading decreases with increase in slenderness ratio of piles at all vertical load levels. The influence of pile slenderness ratio (L/B) on the PVC values observed at a lateral deflection of 0.1B with respect to constant width of pile (B) and varying pile length (L) is shown in Figure 7. Similar results for a pile with constant length of 10 m and varying width (B) are shown in Figure 8. It could be observed that the trends are more or less similar in both the cases.

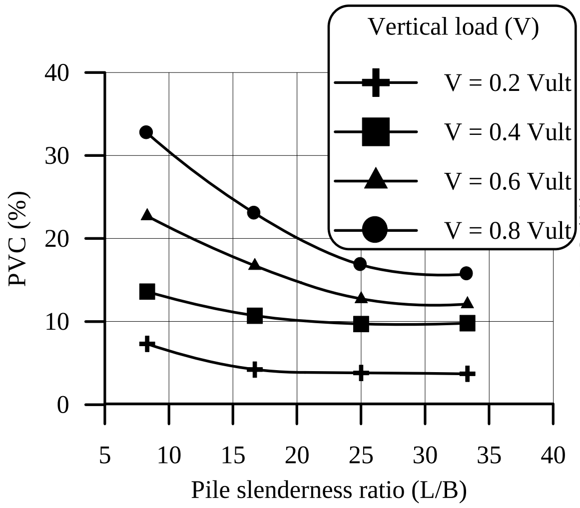


Figure 7. PVC at various L/B with respect to pile length (L)

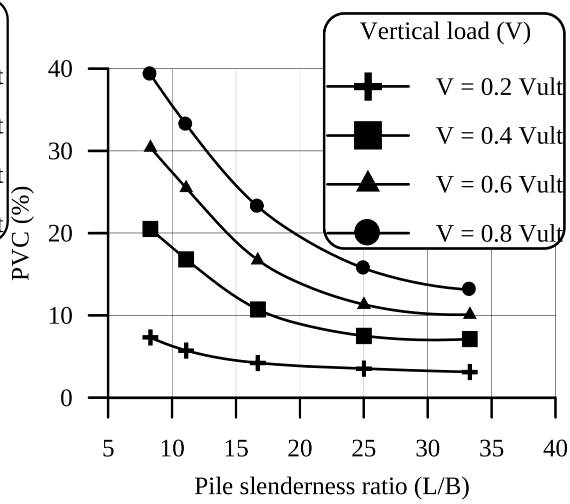


Figure 8. PVC at various L/B with respect to pile width (B)

In general, the presence of vertical load has increased the lateral load capacity at all slenderness ratios. The PVC values remained more or less constant beyond an L/B ratio of 33. As noted above, the influence of vertical loads keeps decreasing as L/B ratio increases. The reason for this can be directly attributed to the reducing intensity of vertical pressure at larger depths due to load dispersion effects. This will in turn lead to lesser changes in confining stresses at larger depths due to vertical load applied at the surface. As the soil strength is related to the operating confining stress, the increase in pile capacity can be expected to be lower for longer piles.

For structural design of piles, the bending moments developed in the pile section are important. The influence of combined loading on the bending moments developed in the pile section has been examined from the results obtained. The bending moments developed in the pile section have been estimated using the well known flexural equation from the vertical stresses (σ_{zz}) developed in the pile section. Figure 9 shows the influence of vertical load on the bending moments developed in the pile section for different L/B ratios and typical vertical load level of $0.6V_{ult}$.

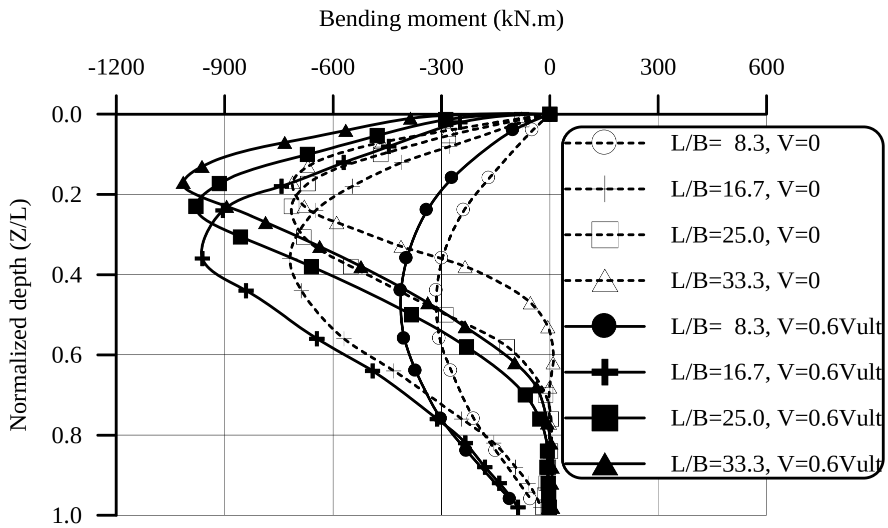


Figure 9. Bending moment developed in the pile section at a deflection of 0.1B

From the figures, the maximum bending moment developed in the pile section is observed to increase with both vertical load levels and L/B ratio. The depth at which the maximum bending moment occurred was influenced very little by the presence of vertical loads. It can also be seen that the point of zero bending moment moved down with increasing vertical load levels for long piles (L/B>16). This result shows that even relatively long piles may behave like short piles installed in the case of sandy soils and subjected to combined loading. The results have made it clear that the design bending moment in piles is higher under combined loading as compared to the piles under pure lateral loading. The exact increase in the bending moment depends very much on the dimensions and properties of the pile, stiffness and strength properties of the soil, and the level of vertical and lateral loads.

CONCLUSIONS

Based on the numerical results from these analyses, the following conclusions can be drawn to investigate the influence of combined loading on the lateral response of free headed piles in sand:

(i) The lateral response of piles under combined loading is influenced by the vertical loads in sandy soils.

(ii) The lateral load capacity of piles in sandy soils increases by as much as 40% with the presence of vertical loads by depending on the level of vertical loads.

(iii) The lateral response of piles under combined vertical and lateral loading is also dependent on the L/B ratio of the pile. As the L/B ratio increases, the influence of combined loading on the lateral capacity reduces. The influence of combined loads remains constant beyond an L/B ratio of 30 in sandy soils

(iv) The design bending moment in the pile section is influenced by the presence of vertical loads. The maximum bending moment increases by as much as 30 to 35% in sandy soils for the range of pile dimensions and soil properties examined in this paper.

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