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Influence of Combined Vertical and Lateral Loading on the Lateral Response of Piles

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ABSTRACT: Pile foundations are often subjected to combined vertical and lateral loads. The current design practices assume that the effect of these two loads is independent of each other. Hence, the pile design is carried out separately for vertical and lateral loads. The laboratory and field test data on the lateral response of piles under the combined action of vertical and lateral loads is rather limited. This paper presents some results from 3D finite element based numerical analyses that show the significant influence of combined loading on the lateral response of piles. The finite element analyses were performed in both homogeneous clay and homogeneous sandy soils. The numerical results have shown that the influence of combined loading on the lateral response of piles is to significantly increase the lateral capacity in sand and marginally decrease the capacity in clayey soils. In general, it was found that the effect of combined vertical and lateral loads in sand is significant even for long piles, which are as long as 30 times the pile width, while in the case of clayey soils, the effect is not significant for piles beyond a length of 15 times the width of the pile. The maximum design bending moments in the laterally loaded piles under combined loading were found to increase with increase in vertical load levels in both the soils.

1 Introduction

Pile foundations are frequently used to support various structures built on loose/soft soils, where shallow foundations would undergo excessive settlements or shear failure. These piles are used to support vertical loads, lateral loads and combinations of vertical and lateral loads. However, in view of the complexity involved in analyzing the piles under combined loading, the current practice is to analyze the piles independently for vertical loads to determine their bearing capacity and settlement and for the lateral load to determine their flexural behaviour. The methods of analysis commonly used in predicting the behaviour of piles and pile groups under pure axial loads could be categorized into: (i) subgrade reaction method (Coyle and Reese 1966; Kraft et al. 1981; Zhu and Chang 2002) (ii) elastic continuum approaches (Poulos 1968; Xu and Poulos 2000), and (iii) finite element methods (Desai 1974; Trochanis et al. 1991; Wang and Sitar 2004). Similarly, the methods to study the behaviour of piles and pile groups under pure lateral loads could be categorized into: (i) limit state method (Broms 1964); (ii) subgrade reaction method (Matlock and Reese 1960); (iii) elastic continuum approach (Poulos 1971; Banerjee and Davis 1978); (iv) p-y method (Reese et al. 1974) and (v) finite element methods (Muqtadir and Desai 1986; Brown and Shie 1991; Trochanis et al. 1991; Kimura et al. 1995; Yang and Jeremic 2002 and 2005).

Studying the interaction effects on piles under combined vertical and lateral loads would no doubt call for a systematic and sophisticated analysis. The literature available in this field is very scanty. The limited information on this aspect based on the analytical investigations (Davisson and Robinson 1965; Ramasamy 1974; Goryunov 1975) reveals that for a given lateral load, the lateral deflection increases with the combination of vertical loads. However, experimental (Pise 1975; Sorochan and Bykov 1976; Jain et al. 1987) and field investigations (McNulty 1956; Bartolomey 1977; Zhukov and Balov 1978) suggest a decrease in lateral deflection with the combination of vertical loads. Anagnostopoulos and Georgiadis (1993) attempted to explain this phenomenon through an experimental model supported by 2-dimensional finite element analysis and reported that the modified status of soil stresses and local plastic volume changes in the soil continuum under combined vertical and lateral loads cannot be accounted for in general by the conventional subgrade reaction, elastic half space and other 2-D approaches and thus a nonlinear 3-D finite element analysis would be the most appropriate approach. Trochanis et al. (1991) attempted to study the axial response of piles with the combination of lateral loads through 3-D finite element analysis. Since the piles are not often adequately designed to resist lateral loads, the response of piles under lateral load in the presence of vertical loads is more critical and interesting for the design engineers. Besides, the influence of pile slenderness ratio (L/B) is also an important parameter to be considered for the design of piles. In view of this, the paper focuses on the study of piles subjected to pure lateral loads and combined vertical and lateral loads through 3D finite element based numerical analyses. The details of the numerical model used in the analysis, the validation of developed model against some field cases and results from parametric studies are discussed in the paper.

2 Finite element based numerical model

All numerical analyses in this investigation were performed by using 3-dimensional geotechnical finite element program GEOFEM3D developed by the authors. The program is supported by a pre-processor to develop 3 dimensional meshes consisting of bar and beam type prismatic elements, 8-node or 20-node isoparametric brick elements, 8 or 16-node zero thickness type interface elements as well as a post-processing tool that is capable of plotting the original mesh, deformed mesh, displacement vectors, extracting nodal displacements and element stresses along a line/selected plane, etc.

2.1 3D finite element mesh details

Figure 1 shows a schematic 3-d finite element mesh for analysis of pile-soil interaction. Based on the symmetry, only half the pile section in the direction of lateral load is analysed (in Figure 1, lateral load is applied along xaxis). 20-node brick elements are used to mesh the pile and the soil continuum. The interface between the pile and the soil has been modelled using 16-node joint elements of zero thickness. All numerical computations of isoparametric elements were performed using reduced numerical integration. The mesh dimensions, number of nodes and elements in the mesh were decided after performing a number of initial trial analyses with several meshes of increasing refinement until the results (displacements and stresses around the pile) did not change significantly with further refinement. The aspect ratios of elements used in the mesh ranged from 0.5 near the pile surface to nearly 5 at the boundaries. The distances to lateral rigid boundaries in the finite element analyses are shown in Figure 1. All the nodes on the lateral boundaries were restrained from moving in the normal direction to the respective surfaces representing rigid, smooth lateral boundaries. All the nodes on the bottom surface were restrained in all the three directions representing rough, rigid bottom surface. Typically, the meshes consisted of approximately 8300 nodes, 1440 20-node brick elements and 52 16-node interface elements.

Figure 1. Typical mesh used for 3D finite element analyses

2.2 Pile-soil details

In the finite element analyses, the pile was treated as a linear elastic material. Von Mises constitutive model with associated flow rule for clayey soils (Yang and Jeremic 2002; Wang and Sitar 2004) and Drucker-Prager constitutive model with non-associated flow rule for sandy soils (Trochanis 1991; Yang and Jeremic 2005) were used to predict the stress-strain behaviour. These models have been preferred in view of their adaptability to define the failure criterion with the use of simple physical properties such as cohesion (c) and friction angle (φ). The failure criterion for the Drucker-Prager model used for sandy soils has the form $F = \alpha J_1 + \sqrt{J_2} - k$, in which J_1 is the first invariant of the stress tensor, J_{2d} is the second invariant of the deviatoric stress tensor and α , k are material constants expressed in terms of the well-known shear strength parameters of soil viz. c and φ. For

clayey soils, α is zero and this equation reduces to $F = \sqrt{\int_{2d} -k}$ representing the Von Mises criterion.

During the plastic flow, the elastic constitutive matrix was first formed based on the current tangent modulus and Poisson's ratio of the material. Then a correction was applied to obtain the elastic-plastic constitutive matrix. The stresses are corrected back to the yield surface along the flow direction (normal to the potential surface defined by the dilation angle (ψ) as described by Nayak and Zienkiewicz (1972).

2.3 Analysis scheme

The finite element analyses were performed in two stages. In the first stage, the in situ stresses were initialized in the soil by performing a dummy analysis using a modified Poison's ratio (μ′) expressed in terms of the at rest earth pressure coefficient K_o as μ' =K_o/(1+K_o). The value of K_o itself was obtained as K_o = 1–sin ϕ . During this stage of analysis, both the pile and the soil elements were assigned the same material properties corresponding to the soil (Young's modulus, Poisson's ratio and unit weight) so as not to generate any extraneous shear stresses. At the end of this stage of analysis, all the deformations and strains are set to zero to define the datum level for further analysis. During the second stage of analysis, the actual properties of the soil and the pile elements were assigned to them. The set of pile-soil properties considered in the analyses are summarized in Table 1.

In the finite element model, the shear strength of the interfaces was defined with zero cohesive strength and $2/3^{rd}$ the friction angle for sandy soils. In the case of clayey soils, the interfaces were assumed to have zero frictional strength and 2/3rd the cohesive strength of the surrounding soil. The interface strength values depend very much on the type of pile material (wood, steel or concrete) and method of installation (driven or bored). The interface strength properties selected for the analyses fall within the range of properties recommended for estimating the skin friction capacity of piles, Bowles (1988). The normal and shear stiffness of the interface elements were initially set to 10⁶ kN/m²/m. These values were decided after performing several analyses with different interface stiffness values. After the shear failure of the interface, the shear stiffness is set to 0.1% of the initial value to permit the relative slip between different materials. The normal stiffness of the interface is set to 0.1% of the initial value when tensile normal stresses develop in order to permit the separation between the pile and the soil.

The external loads were applied in small increments in several load steps with several iterations to satisfy the equilibrium of the system. The iterations were continued at each load step until the norms of out-of-balance force and the incremental displacements were less than 0.5% or until 50 iterations are completed. The analyses were performed using partial Newton-Raphson scheme by updating the stiffness matrix only at the first iteration of each load step.

3 Validation of the numerical model

The validity of the numerical model employed in the program was verified by predicting the pile load test data from two different published cases, one with respect to a long flexible pile under pure lateral load and another for a short rigid pile under combined vertical and lateral loads. The details of these two cases are presented in the following sub-sections.

3.1 Case study – I (Comodromos 2003)

Comodromos (2003) has reported the response of a 52m long, 1 m diameter bored pile under pure lateral loads installed at a bridge site in Greece. The subsoil at the site consists of a thick upper soft silty clay layer with thin layers of loose sand extending to a depth of 36m. Below this, a medium stiff clay layer of 12m thickness existed which is followed by a very dense sandy gravel layer up to the bottom of the borehole. The behaviour of the test pile is analysed by finite element analysis using the program GEOFEM3D. The mesh consisted of 20-node isoparametric continuum brick elements and 16-node zero-thickness joint elements to model the interfaces

between the pile and soil. The properties of various soil layers and the finite element mesh are similar to those reported by Comodromos (2003). The behaviour of soil layers was modelled using the Drucker-Prager constitutive model. Using symmetry, only half of the pile was considered in the analysis. The same sequence of load application used in the field test was followed in the current finite element analysis. The comparison between the finite element predictions and the reported data is shown in Figure 2. It can be seen from the figure that up to a lateral displacement equal to 7% of the pile diameter, the current prediction matches well with both the test data and the numerical result from FLAC 3D program reported by Comodromos (2003). The difference between the current numerical result and that from the FLAC 3D numerical scheme increases to almost 13% at larger displacement level equal to 17.5% of the pile diameter. This difference could be attributed to the difference in the numerical schemes employed and other factors such as the sensitivity of the results to the density of the mesh etc.

Figure 2. Comparison of finite element results with field and numerical data of Comodromos (2003)

3.2 Case study – II (Anagnostopoulos and Georgiadis 1993)

This case study pertains to laboratory model tests on aluminium closed-ended piles of 19 mm outside diameter and 1.5 mm wall thickness, jacked 500 mm into a prepared soft clay bed (w_L = 42%, w_p = 24% and c_u = 28 kN/m²), Anagnostopoulos and Georgiadis (1993). The laboratory tests were performed on a single pile under both vertical and lateral loads applied to the pile head at ground elevation through dead weights. The combined vertical and lateral loads were applied in two stages, in the first stage a vertical load of 160 N was applied and in the second stage the lateral load of 130 N was applied incrementally while the vertical load was kept constant. The Young's modulus (E_s) of the soil was taken as 7500 kPa using the relation E_s \approx 250-400×c_u, (Poulos and Davis, 1980). The Poisson's ratio of the clayey soil was taken as 0.49 assuming an undrained response during the load test. The soil was idealised as a Von Mises material. The pile was modelled as a solid pile with an equivalent modulus to match the flexural rigidity of the hollow test pile. The sequence of the load application used in the current finite element analysis is the same as that followed during the laboratory tests. The comparison between the test data and the predicted results of piles under pure vertical load and combined vertical and lateral loads are shown in Figures 3a and 3b. It could be observed that the comparison is very good both at small and higher load levels. The percentage difference is less than 10% at all load levels for both vertical and lateral response of piles.

The finite element prediction in both the cases matched reasonably well with the test data. Hence, it could be concluded that the numerical scheme adopted in the present investigation is capable of modelling the pile-soil interaction under pure vertical load, lateral load and combination of vertical and lateral loads.

Figure 3. Comparison of present FEM predicted response of pile with experimental test data of Anagnostopoulos and Georgiadis (1993): (a) vertical response of pile, (b) lateral response of pile

4 Parametric studies

A series of 3-d finite element analyses were performed on single free-headed pile in both homogeneous clayey and sandy soils separately. The dimensions of the pile and the soil properties considered in these analyses are reported in Table 1. The response of the piles under pure lateral load was analyzed initially. For studying the response of piles under combined vertical and lateral loads, the influence of vertical load equal to $0.2V_{\text{ult}}$, $0.4V_{\text{ult}}$, $0.6V_{ult}$ and $0.8V_{ult}$ has been considered (where V_{ult} is the ultimate vertical load capacity, evaluated a priori by a separate numerical analysis). The ultimate vertical load (V_{ult}) capacities were estimated as 4000 kN and 1920 kN (corresponding to the point with maximum curvature on the vertical load-settlement response) for clayey soil and sandy soil respectively through analysis of a single pile subjected to pure vertical load.

The combined vertical and lateral loads are applied in two stages. In the first stage, vertical loads were applied and then in the second stage, lateral loads were applied while the vertical load was kept constant. This type of loading is similar to that in field situations like pile jetties, high rise buildings, transmission line towers, overhead water tanks, etc. Here, the piles are first subjected to vertical loading from the weight of the deck or superstructure, etc. The lateral loading may be due to wind, wave loading, ship impact, etc. while the piles are subjected to vertical loads. The analysis in the lateral direction was performed using displacement control (rather than load control) so as to be able to evaluate the lateral loads developed at various lateral displacement levels as a percentage of the size of the pile. The reaction forces developed at the nodes were used to calculate the lateral load corresponding to the applied lateral displacements. The numerical results under pure lateral loads and combined vertical and lateral loads on piles are presented and discussed separately for sandy soils and clayey soils in the following sections. A few analyses were also performed with different slenderness ratios (L/B) of piles to study its influence on piles with combined loading.

4.1 Results and discussions

4.1.1 Influence in sandy soils

Figure 4 shows the influence of vertical load on the lateral response of piles in sandy soils. From the data presented. It is noted that there is a considerable increase in the lateral load capacity under increased vertical load levels. The loads shown in the figure correspond to the symmetric half of the pile section. It can be noted that there is a significant increase in the lateral load capacities, in the order of 7% to 40%, at deflection levels of 0.05B and of the order of 6% to 33% at deflection levels of 0.1B.

The reason for the increase in the lateral capacity under the action of vertical load has been examined through variations in the lateral deflection along the depth of the piles. Figure 5 presents the variation in the lateral deflection along the depth of the pile at a reaction load level of 641 kN (load developed at a deflection of 0.1B under pure lateral load). It can be seen from this figure that the lateral deflections along the depth of the pile was reduced considerably with increasing vertical load levels. Figure 6 shows typical lateral soil stresses in front of the pile at a lateral deflection of 0.1B that are substantially higher at larger vertical load levels. This increase can be directly attributed to the increase in confining stress with increasing vertical loads at different depths caused by the action of vertical load on the pile. This increased confining stress allows for the development of larger lateral and

Figure 4. Lateral load - deflection curves of piles Figure 5. Lateral deflection curves of piles
in sand (deflections at pile bead) piles in sand in sand (deflections at pile head)

Figure 5. Lateral deflection along the depth of

shear stresses along the pile's frictional face as a result of the increased shear strength of the soil. The increase in lateral soil stresses is further examined through the contours of lateral stresses (σxx) around the pile under pure lateral load and with the presence of a vertical load of 0.8Vult in Figure 7a and 7b. These contours are plotted for a lateral deflection equal to 0.1B and at a depth of 2.8 m from the ground surface (the depth where maximum lateral soil stresses occur, Fig. 6). Similarly, the increase in shear stresses (σ_{xy}) over the frictional face of the pile is also examined through the contours of shear stresses around the pile under pure lateral load and with the presence of a vertical load of 0.8Vult in Figure 7c and 7d. These contours are plotted at a lateral deflection of 0.1B and at a depth of 2.8 m from the ground surface (i.e. the depth at which the maximum shear stress occurs). It is clear that the lateral soil stress and the mobilized shear stresses of soil around the pile are higher in the presence of vertical load as compared to the pure lateral load case.

Figure 6. Lateral soil stresses (σ_{xx}) in front of the pile along the length in sandy soil

4.1.2 Influence in clayey soils

Figure 8 shows the lateral load versus lateral deflection response of the piles in clayey soils. The loads shown here correspond to the symmetric half of the pile section. In this case, it is interesting to note a different trend than that observed in the case of sandy soils. In the presence of vertical load, the lateral load developed at all deflection levels is less than the corresponding load developed under pure lateral load. The reduction is not significant for vertical loads up to 0.6Vult. Aubeny et al. (2003) have also reported similar findings in clayey soils through finite element analysis of suction caissons under inclined loads. They have also found that the horizontal anchor load capacity is not significantly affected by vertical components of load for load orientation up to 15°. However, in the present study at higher vertical load level of 0.8Vult, the reduction is significant of the order of 20% of the pile capacity. It is clear that the influence of vertical load is to decrease the lateral load capacity of piles in clayey soils. This reduction in lateral capacity of piles can be attributed to the early failure of interfaces in

Figure 7. Lateral and shear stress contours in xy-plane at 2.8 m depth from ground surface in sands: (a) σ_{xx} contours for pure lateral load, (b) σ_{xy} contours with vertical load of 0.8Vult, (c) contours for pure lateral load (d) σ_{xy} contours with vertical load of 0.8Vult.

Figure 8. Lateral load – deflection curves of piles in clayey soils

the presence of vertical loads. Once the interface between the pile and the surrounding clayey soils fails, further lateral deformation of the pile will not result in increased lateral stresses in the soil around the pile. This reduction in lateral soil stresses is illustrated in Figure 9 that shows the contours of lateral soil stresses (σ_{xx}) at a lateral deflection of 0.1B and at a depth of 2.8 m from the ground surface for pure lateral loading and with a vertical load of 0.8V_{ult}. This reduction in lateral soil stresses (σ_{xx}) will lead to the development of lower resistances to lateral pile deformation in the presence of a vertical load.

Figure 9. Lateral stress contours in xy-plane at 2.8 m from ground surface in clayey soils: (a) σ_{xx} contours under pure lateral load (b) σ_{xx} contours with vertical load of 0.8V_{ult}

4.1.3 Influence of Pile Slenderness Ratio (L/B)

The influence of pile slenderness ratio (L/B) under combined vertical and lateral loading was studied by performing a 3-dimensional finite element analysis of 600×600 mm size square pile with different lengths of 5, 10, 15 and 20 m. Similarly, 3-dimensional finite element analyses were also performed by keeping the pile length constant at 10 m and varying the pile widths to 300, 400, 600, 900 and 1200 mm. All these analyses were performed separately for both clayey soils and sandy soils in the same manner as described earlier. Based on the numerical results obtained, the **P**ercentage **V**ariation in lateral **C**apacity (PVC) with respect to different levels of vertical loads is calculated for various L/B ratios with respect to various pile length (L) and widths (B) considered in the analysis. The PVC is defined as follows in terms of the **L**ateral load **C**apacity **W**ith **V**ertical load (LCWV) and the **L**ateral load **C**apacity under **P**ure **L**ateral loading (LCPL),

$$
PVC = \frac{LCWV - LCPL}{LCPL} \times 100\%
$$
 (1)

The results have shown that the influence of a vertical load decreases with increase in slenderness ratio of piles at all vertical load levels for both types of soils. In general, it was found that the influence of vertical load with respect to pile slenderness ratio (L/B) was found to be more in the case of sandy soils than in clayey soils. The influence of pile slenderness ratio (L/B) on the PVC values observed for both types of soils at a lateral deflection of 0.1B with respect to constant width of pile (B) and varying pile length (L) is shown in Figure 10. Similar results for a pile with constant length of 10 m and varying width (B) are shown in Figure 11. It could be observed that the trends are more or less similar in both the cases.

Figure 10. Percentage Variation in lateral Capacity (PVC) at various L/B with respect to pile length (L)

Figure 11. PVC at various L/B with respect to pile width (B)

In the case of sandy soils, the presence of a vertical load has increased the lateral load capacity at all

slenderness ratios. However, in the case of clayey soils, the influence of a vertical load is observed only at small L/B ratio of 8.3. The PVC values remained more or less constant beyond an L/B ratio of 16.7 in the case of clayey soils. On the other hand, in the case of sandy soils, the PVC values are dependent on both L/B value and the level of vertical load. The PVC value was found to be dependent even up to L/B value of 33 in the case of sandy soils. As noted above, the influence of vertical loads keeps decreasing as L/B ratio increases. The reason for this can be directly attributed to the reducing intensity of vertical pressure at larger depths due to load dispersion effects. This will in turn lead to lesser changes in confining stresses at larger depths due to vertical load applied at the surface. As the soil strength is related to the operating confining stress, the increase in pile capacity can be expected to be lower for longer piles. This aspect is examined through the Percentage Increase in lateral soil Stresses (PIS) in front of the pile for various L/B ratios. A quantity termed "PIS" is defined as follows as the Lateral soil Stress (σxx) With the presence of Vertical load (LSWV) as compared to the corresponding Lateral soil Stress under Pure Lateral loading (LSPL).

$$
PIS = \frac{LSWV - LSPL}{LSPL} \times 100\%
$$
 (2)

Figure 12 shows typical PIS values for various L/B ratios at vertical load level of $0.8V_{ult}$ and lateral deflection of 0.1B. The influence of L/B on PIS values can be clearly seen in the figure. The PIS values increased considerably and over longer lengths for shorter piles as compared to longer piles.

Figure 12. **P**ercentage **I**ncrease in lateral soil **S**tresses (PIS) in front of the pile at different L/B ratios

For structural design of piles, the bending moments developed in the pile section are important. The influence of combined loading on the bending moments developed in the pile section has been examined from the results obtained. The bending moments developed in the pile section have been estimated using the well known flexural equation from the vertical stresses (σ_{zz}) developed in the pile section. Figures 13a and 13b show the influence of vertical load on the bending moments developed in the pile section for different L/B ratios and typical vertical load level of 0.6V_{ult}.

From the figures, the maximum bending moment developed in the pile section is observed to increase with both vertical load levels and L/B ratio. The depth at which the maximum bending moment occurred was influenced very little by the presence of vertical loads in both clayey and sandy soils. In the case of sandy soils, the point of zero bending moment moved down with increasing vertical load levels for long piles (L/B>16). On the other hand, the point of zero bending moment is not much affected by the presence of vertical loads in the case of clayey soils for all lengths of piles. This result shows that even relatively long piles may behave like short piles in the case of sandy soils and subjected to combined loading. The results have made it clear that the design bending moment in piles is higher under combined loading as compared to the piles under pure lateral loading. The exact increase in the bending moment depends very much on the dimensions and properties of the pile, stiffness and strength properties of the soil and the level of vertical and lateral loads.

Figure 13. Bending moment developed in the pile section at 0.1B displacement: (a) in sandy soils (b) in clayey soils

5 Conclusions

Three-dimensional finite element based numerical analyses have been carried out to investigate the lateral response of single pile under combined vertical and lateral loading. Based on the numerical results from these analyses, the following conclusions can be drawn:

- \triangleright The lateral response of piles under combined vertical and lateral loading is influenced by the vertical loads in both clayey and sandy soils.
- \triangleright The lateral load capacity of piles in sandy soils increases by as much as 40% with the presence of vertical loads depending on the level of vertical loads.
- \triangleright The presence of a vertical load marginally reduces the lateral load capacity of piles in clayey soils for vertical load level up to $0.6V_{\text{ult}}$ and by as much as 20% for higher vertical load levels.
- \triangleright The lateral response of piles under combined vertical and lateral loading is also dependent on the L/B ratio of the pile. As the L/B ratio increases, the influence of combined loading on the lateral capacity reduces. The influence of combined loads remains constant beyond an L/B ratio of 25 in sandy soils and 16 in clayey soils.
- \triangleright The design bending moment in the pile section is influenced by the presence of vertical loads. The maximum bending moment increases by as much as 30 to 35% in sandy soils for the range of pile dimensions and soil properties examined in this paper. In the case of clayey soils, the maximum bending moment increases by about 10 to 15% for L/B values less than 15 and about 30% for longer piles.

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