A New Approach for Damage Identification in Multi Storey Shear Structure from Sparse Modal Information

S.K.Panigrahi*, S.Chakraverty and B.K.Mishra*****

*Scientist, B.P.P.P. Divn., Central Building Research Institute, Roorkee, India
e-mail : panigrahi1111@yahoo.com
** Professor, Department of Applied Mathematics, National Institute of Technology, Rourkela, India,e-mail :sne_chak@yahoo.com
(On Lien from : Central Building Research Institute, Roorkee)
*** Professor, Mechanical and Industrial Engineering Department,Indian Institute of Technology Roorkee, Roorkee-247 667, India,e-mail : bhanufme@iitr.ernet.in

Abstract

This paper presents a procedure to identify the damage of a multi storey shear structure using residual force vector method along with Genetic Algorithm (GA) from sparse modal information. In system identification of structural problems, a large number of degrees of freedom in their finite element models are required. It is difficult and expensive to measure modal response at many locations. A technique has been first described to compute the full mode shape from partial mode shape for a particular frequency. Then the concept of the residual force vector is used to specify an objective function using the computed modal information. Finally GA has been implemented for optimizing this function to get the damage factors. Experimental data are simulated numerically by solving eigen value problem of the damaged structure with inclusion of random noise on the vibration characteristics. Reliability of the procedure has been shown by various examples of multi storey structure with different known modal information.

KEY WORDS : Shear Structure, Residual Force, Genetic Algorithm, Damage, Noise level

INTRODUCTION

During the last three decades, vibration based methods have been developed and applied to detect structural damage in the civil, mechanical and aerospace engineering [1-2]. Identifying the structural damage with the measured vibration data is an inverse approach in mathematics. The usual damage detection methods minimize an objective function, which is defined in terms of the discrepancies between the vibration data identified by modal testing and those computed from the analytical model [3,4]. These conventional optimization methods are gradient based and usually lead to a local minimum only.

In the last two decades, since first introduced by Holland [5], Genetic Algorithm (GA) has been widely applied to various optimization problems [6]. Many authors have recently taken up this optimization problem using Neural Networks, Genetic Algorithm and Neural Network with GA [7.8.9] by studying the variation of localized damage as a function of modal test data and machine learning. More recently, the residual force concept has received increased attention as regards application to damage detection and assessment [7,10]. When the objective function from residual force concept is optimized by GA, it gives good results with noise polluted experimental data. As such, Mares and Surace [8] and Rao et.al. [11] have used this concept for damage identification in truss structures and beams along with GA. Panigrahi et. al.[12] have used this concept of damage identification in a shear structure. Also they used this for damage identification of a tapered and non-homogeneous beams [13,14]. In these papers, the authors have used full mode shape of all modes that have been considered in the objective function formulation.

Sparsity of measured data are inevitable in testing engineering structures. Sparsity of measurements creates problem in identification of structures/ structural members. Most structures require a large number of degrees of freedom in their finite element models due to their size and complexity. But in practice, it is difficult and expensive to measure modal displacements at many locations. So, only a small subset of all the degrees of freedom in the model is normally measured. There are number of schemes available in literature to overcome the difficulty due to sparseness [15, 16, 17, 18,19 and 20]. Mares and Surace [8] in their model for damage identification in a beam structure condensed the FE model by using the IRS-method (Improved Reduced system). Yuan et al [21] have developed a method that estimates mass and stiffness matrices of shear structure from first two orders of structural mode measurement. Chakraverty [22] proposed computationally efficient procedures to refine the methods of Yuan et al. [21] to identify the structural parameters from modal test data. He obtained the full mode shape by knowing the first and top floor modal displacement at a particular frequency. Chakraverty [22] has used Holzer criteria along with other numerical techniques in solving the above problems. Medhi et.al. [23] in their work utilise system identification technique for health monitoring of shear building, wherein parametric state space modeling has been adopted.

In the present work, the case of lumped mass systems such as shear building is considered for damage identification with sparse modal information. First a method is being described to obtain the complete mode shape at a particular frequency from partial modal information. In fact, full mode shape of a particular frequency may be obtained by simply using the known frequency value.

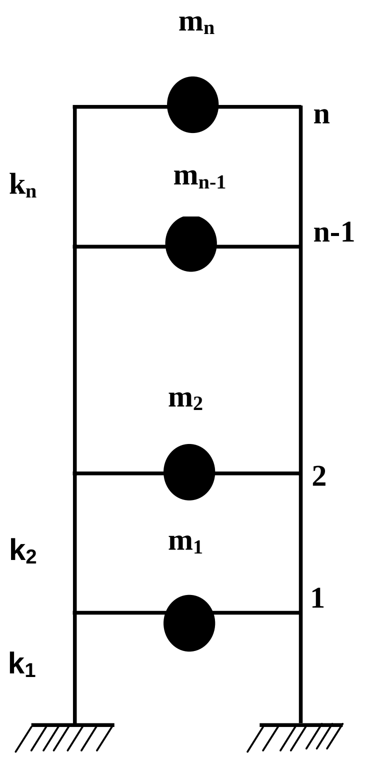

Fig.1. Multistorey structure with n levels.

Next, this method is used for a damaged structure in finding the full mode shape of each mode in terms of damage factors. An objective function is developed using residual force vector considering different number of modes and corresponding mode shape obtained as above. This objective function is optimised by GA to obtain the damage factors. The algorithm is tested by taking different examples of multistorey structure with a damaged situations using different number of modal information. Noise is introduced in both frequency and mode shape parameter and the model is validated. As such here this study has been done to arrive at minimum number of modal information that may be required for better damage identification. Accordingly different problems are discussed.

PROBLEM FORMULATION

Computation of full modal information from partial modal data

The methodology for computing unknown modal data corresponds to a particular frequency using known partial mode shape is explained. A highly idealised n-storey shear structure is considered as shown in Figure 1. For a shear structure of n levels, with k_1, k_2..... k_n and m_1 , m_2......m_n as the corresponding stiffness and mass of different levels, the equation of motion of free vibration may be written as [25]

$$\left(K - \lambda_j M\right)\begin{Bmatrix} \phi_j^{(1)} \\ \phi_j^{(2)} \\ \vdots \\ \phi_j^{(n)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \qquad (1)$$

Where M and K are the global mass and stiffness matrices
$\lambda_j=\omega_j^2$ is the j^{th} eigen value and $\phi_j^{(r)}$ where $r=1, n$ designates the j^{th} mode shape or eigen vector.
The equation (1) may be alternatively expressed as

$$[A]_{nXn}\{\phi_j\}_{nX1} = \{0\} \qquad (2)$$

Here n is the total number of levels of the shear structure. Suppose modal response is not measured at all the levels but only at some of the levels. Let total number of levels where modal measurements are made be Q. It means that we know the modal response of a total of Q levels at the j^{th} frequency. The displacement at other levels may be calculated analytically from the existing modal data by rearranging equation (2). The levels whose modal component are known may be represented by P_q where q varies from 1 to Q.
Now, the above equation may be rewritten as

$$[A']_{(n-Q)X(n-Q)}\ \{\phi_j'\}_{(n-Q)X1} = \{Z\} \qquad (3)$$

where A' is the reduced A matrix obtained by eliminating P_q^{th} rows and P_q^{th} columns. Similarly ϕ_j' is the reduced column matrix ϕ_j without P_q^{th} elements.
A_{Pq}' is the P_q^{th} column matrix of A without the P_q^{th} elements in it.
ϕ_j' represents the unknown modal matrix. This may be computed from equation (3) as below

$$\{\phi_j'\} = (A'^T\ A')^{-1}\,(A'^T\ \{Z\}) \qquad (4)$$

DAMAGE IDENTIFICATION

For the damage identification, two cases are investigated. In the first case, no mode shape information is available and in the second case two modal measurements are known at any two levels. For each case, first the unknown mode shape information at a given frequency is calculated following the method of the previous section. In a damaged structure, the stiffness values are not known. Hence, stiffness values of each floor are multiplied by corresponding damage factors i.e.β_i $(i = 1,2\ldots n)$. The unknown modal information is obtained in terms of the damage factors. This full modal information is then used to develop the fitness function.

First case : Only frequency value is known

Here, only the natural frequencies are known and no modal displacement at any level is known. The mode shape may be assumed to be normalized with respect to modal response at l^{th} level. Here, we may assume modal data at any level as unity say at l^{th} level it is unity. The equation (3) may be written as

$$\begin{bmatrix} P_1 & -k_2\beta_2 & 0 & 0 & 0 & 0 & & 0 & & \\ 0 & & & & & & & \vdots & & \\ 0 & & -k_{l-1}\beta_{l-1} & P_{l-1} & 0 & 0 & & 0 & & \\ 0 & & \cdots & 0 & 0 & P_{l+1} & -k_{l+2}\beta_{l+2} & 0 & & \\ 0 & & \cdots & & \cdots & & & \vdots & & \\ 0 & & \cdots & 0 & \cdots & & & \vdots & & \\ 0 & & \cdots & & \cdots & 0 & & \vdots & & \\ 0 & & \cdots & & \cdots & & & \vdots & & \\ 0 & & \cdots & 0 & \cdots & 0 & & \vdots & -k_n\beta_n & k_n\beta_n - \lambda_j m_n \end{bmatrix}$$

$$X \quad \begin{Bmatrix} \phi_{jd}^{(1)} \\ \vdots \\ \phi_{jd}^{(l-1)} \\ \phi_{jd}^{(l+1)} \\ \vdots \\ \vdots \\ \vdots \\ \phi_i^{(n)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \vdots \\ k_l\beta_l\phi_{jd}^{(l)} \\ k_{l+1}\beta_{l+1}\phi_{jd}^{(l)} \\ \\ \vdots \\ \vdots \\ 0 \end{Bmatrix} \qquad (5)$$

$\phi_{jd}^{(l)} = 1$

i.e. $[A']\{\phi_{jd}\}=\{Z\}$, where $[A']$ is again a square matrix on the left hand side, $\{\phi'_{jd}\} = \{\phi_j^{(1)}, \phi_j^{(2)}, ..\phi_j^{(l-1)}, \phi_j^{(l+1)} ...., \phi_j^{(n)}\}^T$ and $\{Z\}$ is the column vector on the right-hand-side of above equation (5) respectively. Similar to the earlier cases, the modal data for all unknown levels ϕ_{jd} may be obtained using equation (4), i.e.

$$\left\{\phi'_{jd}(\beta_1, \beta_2 ... \beta_n\right\} = \left(A'^T\ A'\right)^{-1}\left(A'^T\ Z\right) \qquad (6)$$

Here, it is assumed that the natural frequencies and mode shapes of the damaged structure continue to satisfy the eigen value.

The residual force vector for j^{th} mode in terms of β_i can be written as,

$$R_j(\beta_1, \beta_2, ...\beta_n) = -\lambda_{jd}[M]\{\phi_{jd}(\beta_1, \beta_2, ...\beta_n)\}$$
$$+ K_d(\beta_1, \beta_2, ...\beta_n)\left\{\phi_{jd}(\beta_1, \beta_{2...}\beta_n)\right\} \qquad (7)$$

Here R_j is a function of β_i. R_j will be a null matrix, only if a correct set of β_i and correct values of damaged modal information λ_{jd} and ϕ_{jd} for the j^{th} mode of vibration are substituted in equation (7).

Then the objective function in terms of damage factors may be written as,

$$f(\beta_1,\beta_2,....\beta_n)=\sum_{j=1}^{r} R_j(\beta_1,\beta_2,....\beta_n)^T R_j(\beta_1,\beta_2,....\beta_n) \qquad (8)$$

where n is the number of stories and r is the number of modes taken into consideration.

β_i may be obtained by minimising the objective function. But as GA deals with maximization problems, it is necessary to specify a modified version of the objective function to be maximized. The fitness function V for the present task is defined as,

$$V(\beta_1,\beta_2,.........\beta_n)= C_1 \Big/ \left\{C_2 + f(\beta_1,\beta_2,.........\beta_n)\right\} \qquad (9)$$

where C_1 represents a constant used to control the value of the objective function, C_2 represents a constant used to build a well-defined function for the ideal case (i.e. with no experimental error). The values of C_1 and C_2 are taken as unity in this study.

Second case: Frequency value along with corresponding partial mode shape is known

Let us first suppose that two modal measurements are made at r^{th} and s^{th} levels of the damaged structure and so, *1*st to *(r-1)*th, *(r+1)*th to *(s-1)*th and *(s+1)*th to n^{th} level modal components are unknown. Now equation (3) may be written for n storey structure including damage factors as,

$$\begin{bmatrix}
P_1 & -k_2\beta_2 & 0 & 0 & ... & 0 & ... & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & & & ... & & ... & & & ... & & & ... & & \vdots \\
0 & 0 & 0 & -k_{r-1}\beta_{r-1} & P_{r-1} & ... & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & ... & 0 & 0 & P_{r+1} & -k_{r+2}\beta_{r+2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & ... & 0 & ... & & & ... & & & ... & & \vdots \\
0 & 0 & 0 & ... & 0 & ... & & & ... & & & ... & & \vdots \\
0 & 0 & 0 & ... & 0 & ... & 0 & 0 & 0 & P_{s+1} & -k_{s+2}\beta_{s+2} & 0 & 0 & 0 \\
0 & 0 & 0 & ... & & ... & & & \vdots & & & ... & & \vdots \\
0 & 0 & 0 & ... & & ... & 0 & 0 & \vdots & 0 & 0 & 0 & -k_n\beta_n & k_n\beta_n-\lambda_j m_n
\end{bmatrix}$$

$$X\begin{Bmatrix} \phi_{jd}^{(1)} \\ \vdots \\ \phi_{jd}^{(r-1)} \\ \phi_{jd}^{(r+1)} \\ \vdots \\ \phi_{jd}^{(s-1)} \\ \phi_{jd}^{(s+1)} \\ \vdots \\ \phi_{jd}^{(n)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \vdots \\ k_r\beta_r\phi_{jd}^{(r)} \\ k_{r+1}\beta_{r+1}\phi_{jd}^{(r)} \\ 0 \\ k_s\beta_s\phi_{jd}^{(s)} \\ k_{s+1}\beta_{s+1}\phi_{jd}^{(s)} \\ \vdots \\ 0 \end{Bmatrix} \qquad (10)$$

Each of the above matrix equation (10) may be written in compact form as $[A'][\phi_{jd}]=\{Z\}$, where $[A']$ is the square matrix on the left hand side, $\{\phi'_{jd}\} = \{\phi_{jd}^{(1)},...\phi_{jd}^{(r-1)},\phi_{jd}^{(r+1)},..\phi_{jd}^{(s-1)},\phi_{jd}^{(s+1)}...,\phi_{jd}^{(n)}\}^T$ and $\{Z\}$ is the column vector on the right-hand-side. Equation (5) may be written as,

$$\left\{\phi'_{jd}(\beta_1,\beta_2...\beta_n\right\} = \left(A'^T\ A'\right)^{-1}\left(A'^T\ Z\right). \tag{11}$$

The fitness function may be obtained in a manner as discussed for the first case.

After some trials, the GA parameters for the present work were set up as:
Population Size: 20, Crossover probability: 0.70, Mutation probability: 0.005
Each structural parameter β_i was represented as a 10-bit binary numbers with variable limits 0 to 1.

ILLUSTRATIVE EXAMPLES

Here, a 8-storey shear structure is considered. The structural characteristics are given in Table 1. Forth and seventh storey of the structure are assumed to be partially damaged to an extent of 45% and 30% respectively. Alternatively it means that the damage factors for forth and seventh storey of the structure are 0.55 and 0.7 respectively.

Table 1. The design values of stiffness and mass for eight storey structure

Stiffness	**k1**	**k2**	**k3**	**k4**	**k5**	**k6**	**k7**	**k8**
Values in kN/m	108	54	54	54	45	40.5	36	36
Mass	**m1**	**m2**	**m3**	**m4**	**m5**	**m6**	**m7**	**m8**
Values in 1000Kg.	90	36	36	36	27	27	22.5	21.6

The input modal data is generated from the vibration analysis of the above shear structure. For the said problem, global stiffness matrix is computed by reducing the stiffness of the fourth and seventh storey by 45% and 30% respectively. From global mass and stiffness matrices, eigen value problem is solved. The frequency and mode shape information are given as input to the damage identification model. At first, the measurement noise is ignored. Then for simulating noise polluted experimental measurement, a random noise of 2% in frequency and 5% in mode shape has been imposed.

The objective function has been developed from the simulated modal data using different numbers of frequencies. The number of modal frequencies considered is varied from two to eight. It is assumed that for same frequency value different number of modal components are known. First, it is assumed that only the frequency value of each mode is known and mode shape is completely unknown. Then it is assumed that two modal components at a particular frequency value are known. Then number of known modal components is gradually increased. For each case the unknown modal information in terms of damage factors is computed and the full mode shape is used in the formation of objective function. The objective function is optimized to get the damage factors.

For the above problem, the comparison of actual damage factors with the identified damage factors using different number of modes are shown in Table 4 when only frequency values are known. It is assumed that frequency values are noise polluted. Table 2 also shows the damage prediction in the presence of noise.

From the Table 2, it is observed that in the absence of noise damage can be identified with a maximum error of around 10% using six modes. It may be noted as usual that by increasing number of modes, the accuracy can be increased. With the considered noise level damage can be identified using eight modes with an error of 11%.

Table 2 : Damage factors obtained using only the frequency values for different number of modes (without noise and with noise)

Damage Facto	**Theoretical values**	**Identified values without noise**				**Identified Values wit noise level II**			
		Different Number of Modes				**Different Number of Modes**			
		2	4	6	8	2	4	6	8
β_1	1.0	0.801	0.984	0.933	0.948	0.712	0.811	0.890	0.921
β_2	1.0	0.861	0.860	0.998	0.959	0.745	0.843	0.863	0.918
β_3	1.0	0.801	0.856	0.916	1.000	0.927	0.909	0.859	0.932
β_4	0.55	0.450	0.597	0.583	0.560	0.664	0.601	0.561	0.531
β_5	1.0	0.950	0.881	0.917	1.000	0.959	0.959	0.859	0.890
β_6	1.0	0.891	0.957	0.996	0.929	0.881	0.881	0.891	0.960
β_7	0.7	0.774	0.772	0.769	0.789	0.665	0.765	0.789	0.669
β_8	1.0	0.823	0.916	0.900	0.939	0.863	0.863	0.901	0.902

When sensors are placed at first and second level, the results obtained with various modes are shown in Table 3.

Table 3 : Damage factors obtained using the frequency value and corresponding partial mode shape at first and second level for different number of modes (without noise and with noise)

Damage Factor	**Theoretical values**	**Identified values without noise**				**Identified Values with noise**			
		Different Number of Modes				**Different Number of Modes**			
		2		6	8	2	4		8
β_1	1.0	0.851	0.998	0.943	0.988	0.802	0.811	0.900	0.956
β_2	1.0	0.891	0.890	0.976	0.966	0.745	0.843	0.899	0.938
β_3	1.0	0.880	0.889	0.936	0,989	0.927	0.909	0.909	0.945
β_4	0.55	0.490	0.598	0.523	0.541	0.601	0.570	0.579	0.534
β_5	1.0	0.941	0.899	0.932	0.987	0.959	0.959	0.891	0.940
β_6	1.0	0.867	0.949	0.999	0.951	0.881	0.881	0.893	0.956
β_7	0.7	0.731	0.750	0.735	0.728	0.679	0.752	0.742	0.719
β_8	1.0	0.860	0.902	0.932	0.962	0.863	0.863	0.921	0.933

From the above table, it is observed that in the absence of noise damage can be identified with a maximum error of around 7% using six modes. Again by increasing number of modes, the accuracy increases. Damage can be identified using six modes with an error of 11% when the noise level as specified is considered. Inclusion of ten modes in the analysis, the error falls to 5%.

Table 4 shows the damage factors with frequency vales and corresponding known partial mode shape at first and eighth level.

Table 4 : Damage factors obtained using the frequency value and corresponding partial mode shape at first and eighth level for different number of modes (without noise and with noise)

Damage Factor	**Theoretical values**	**Identified values without noise**				**Identified Values with noise**			
		Different Number of Modes				**Different Number of Modes**			
			4	6	8	2	4	6	8
β_1	1.0	0.841	0.987	0.953	0.995	0.891	0.881	0.952	0.961
β_2	1.0	0.901	0.920	0.976	0.989	0.815	0.876	0.912	0.979
β_3	1.0	0.891	0.912	0.964	0,987	0.827	0.918	0.918	0.965
β_4	0.55	0.498	0.531	0.541	0.542	0.598	0.597	0.532	0.557
β_5	1.0	0.891	0.819	0.978	0.989	0.912	0.939	0.961	0.971
β_6	1.0	0.898	0.949	0.979	0.980	0.901	0.911	0.987	0.957
β_7	0.7	0.721	0.727	0.733	0.709	0.739	0.726	0.689	0.721
β_8	1.0	0.871	0.932	0.961	0.978	0.887	0.887	0.921	0.959

From the above table, it is observed that in the absence of noise damage can be identified with a maximum error of around 5% using six modes. Similar trend of accuracy may be seen by increasing the number of modes. Here also by considering noise level as mentioned, damage can be identified using six modes with an error of 9% and the error falls to 4% with eight modes.

Lastly, Figure 2 shows the best fitness and average fitness values versus the number of iterations for the case with six modes and corresponding known first and last level modal data with noise. The best fitness value has been obtained here at 602 iteration number.

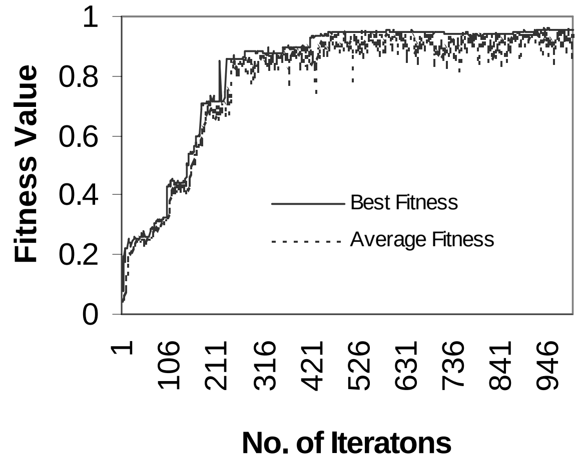

Fig.2 : Best fitness value and average fitness value vs. No. of iterations

DISCUSSION

In this study, an approach for detecting damage based on residual force vector method using GA has been presented. One of the main advantages of the GA approach is its easy implementation, relying on forward analysis. Furthermore, unlike many classical methods, there is no need for computation of derivatives and no initial guess about the structural parameters is needed. Of specific importance in practical applications is that the new approach has exhibited greater robustness in numerical simulations of structural systems, particularly the influence of noise on the experimental data on the effectiveness of the identification procedure.

Moreover method has also been described to get the full mode shape from partial mode shape. The methodology is used along with residual force vector method and GA to get the damage factors from sparse modal data. From the tables it is observed that when there is no noise on simulated data the accuracy in getting the damage factors is better than those with noise on modal data. By increasing the number of known modes, the accuracy increases. Also the error in the computation when only frequency value is known for getting damage factors is more than those obtained with simultaneous knowledge of frequency value with first and second level modal data. Again the accuracy depends upon number of known modal information and it increases as we take more known data. It is also observed that the accuracy is better using known modal data of first and last level than by using the known modal data at 1^{st} and 2^{nd} level. This may be due to the less computational error in finding unknown mode shape knowing the first and second level data then first and last level data.

CONCLUSIONS

A procedure has been presented for the simultaneous location and quantification of the damage in shear structure with sparse modal information. Genetic algorithm has been employed for which the optimization function has been formulated in term of modified residual force vectors. Investigation has also been done to study the said problem with and without noise polluted experimental data. Minimum number of modes and corresponding partial modal information required to get the best result has been discussed. The damage factors identified for the problem, which are obtained by using GA for optimization purpose, show excellent agreement with those chosen for the mechanical simulation of these damaged structures.

ACKNOWLEDGEMENT

The first author would like to thank Director, Central Building Research Institute, Roorkee for giving permission to present and publish this paper.

REFERENCES

1. Cawley P, Adams RD. (1979), “The Location of Defects in Structures from Measurement of Natural Frequencies.” J. Strain.Anal. 14(2): 49-57.
2. Stubbs N, Osegueda R. (1990), “Global Damage Detection in Solids: Experimental Verification.” Int. J. Anal. Exp. Modal Anal. 5(2): 81-97.
3. Hajela P, Soeiro FJ. (1990) ‘Structural Damage Detection Based on Static and Modal Analysis.” AIAA J. 28(4): 1110-1115.
4. Chakraverty S, Singh VP, Sharma RK. (2006) ‘Regression Based Weight Generation Algorithm in Neural Network for Estimation of Frequencies of Vibrating Plates.” Computer methods Appl. Mech. Engg. 195: 4194-4202.
5. Holland JH. (1975) “Adaption in Natural and Artificial Systems.” University of Miichigan, Ann Arbor, Michigan.USA,.
6. Goldberg DE. (1989) “Genetic Algorithms in Search, Optimization and Machine Learning.” Addsion-Weesley Longman: Reading, Mass.
7. Tsou P, Shen MHH. (1994) “Structural Damage Detection and Identification using Neural Networks.” AIAA J; 32:176-183.
8. Mares C, Surace C. (1996) “An Application of Genetic Algorithm to Identify Damage in Elastic Structures.” J. Sound Vib. 195: 195-215.
9. Marawala T, Chakraverty S. (2006) “Fault Classification in Structures with Incomplete Measured Data using Auto associative Neural Networks and Genetic Algorithm.” Current Science ; 90(4): 542-548.
10. Chakraverty S. (2007) “Identification of structural parameters of two storey shear buildings by the iterative training of Neural Networks” Architectural Science Review; 50.
11. Rao M Anand, Srinivas J, Murthy BSN. (2004) “Damage Detection in Vibrating Bodies using Genetic Algorithms.” Computers and Structures 82: 963-968.

12. Panigrahi S.K., Chakraverty S. and Mishra B.K. (2008) “Damage identification in vibrating lumped mass systems using genetic algorithm.” The Sixth Structural Engg. Convention, SERC, Chennai, 18-20, p331-342.
13. Panigrahi SK, Chakraverty S, Mishra BK. (2008) “Damage Identification in Structural Members with Variable Thickness using Genetic Algorithm.” International journal of Engineering Simulation 8(3):3-10.
14. Panigrahi SK, Chakraverty S, Mishra BK. (2008) “Application of Genetic Algorithm for Damage Identification of Non-homogeneous Tapered Beam.” Computer Methods in Material Science; 8(2): 93-102.
15. Hoff C. (1989) “The use of reduced finite element methods in system identification.” Earthquake Engg. and Structural Dynamics, 18, 875-887.
16. Kammer D.C., Jensen B.M. and Mason D.R. (1989) “Test analysis correlation of the space shuttle solid rocket motor center segment.” Journal of Spacecraft, 26, 266-273.
17. Guyan R.J. (1965), ‘Reduction of stiffness and mass matrices.” AIAA Journal, 3(2), 380.
18. Smith S.W. and Beattie C.A. (1990) “Simultaneous expansion and orthogonalisation of measured modes for structure identification.” AIAA Dynamics Specialist Conference, Long Beach, CA, 261-270, April 5-6.
19. Kim H.M. and Bartkowicz T.J. (1993) “Damage detection and health monitoring of large space structures.” 34th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Material Conference, La Jolla, 3527-3533.
20. Pothisiri T and Hjelmstad K.D. (2003) “Structural damage detection and assessment from modal response.” Journal of Engg. Mechanics, 129(2), 135-245.
21. Yuan P, Wu Z and Ma X. (1998) “Estimated mass and stiffness matrices of shear building from modal test data.” Earthquake Engineering and Structural Dynamics, 27, 415-421.
22. Chakraverty S. (2005) “Identification of structural parameters of multistory shear buildings from modal data. Earthquake Engineering and Structural Dynamics.” 34, 543-554.
23. Medhi Manab, Dutta Anjana and Deb S. K. (2007) “Health monitoring of multistoried building using parametric state space modeling.” Smart Systems and Structures, 4(1).
24. Michalewicz Z. (1994) “Genetic Algorithms + Data Structure = Evolution Programs.” Spring-Verlag: Second, Extended edition, Berlin.
25. Chopra A.K. (2004) “Dynamics of structures, Theory and applications to earthquake engineering.” Prentice-Hall of India Pvt. Ltd., New Delhi.