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IDENTIFICATION OF STRUCTURAL PARAMETERS OF MULTISTOREY SHEAR STRUCTURES FROM MINIMUM NUMBER OF MODAL DATA

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ABSTRACT

This paper presents a procedure to identify the location and extent of damage in a multi storey structure from minimum number of modal information. Earlier studies include the all or a part of the total frequencies and their full modal data for the identification of the structural damage. A method has been proposed here to estimate all the components of the mode shape of a particular order of frequency by knowing the modal test data of ground level of the structure. The reliability of the procedure has been shown by various examples of multistorey structure.

KEY WORDS: Identification, Structure, Stiffness, Mass, Frequency Parameter, Mode shape

INTRODUCTION

The modelling in structural dynamics problems may be categorized as direct or inverse problem. The direct problem consists of finding the response for a specified input or excitation. In the inverse problem, first the response is known then to develop an accurate mathematical model of the system, which is known as the system identification. This involves the determination of the models and the estimation of values of structural parameters using measured data. Study of system identification technique for knowing the actual states of engineering structures have received much attention in recent years. This is because of the well-known fact that the full-scale experimental studies are more expansive and also in some cases difficult to perform. Intelligent mathematical/computational algorithms decrease the instrumentation cost in general and also these are important, as full-scale instrumentation is somewhat difficult in certain structures.

Although, few researchers have studied the above issues but, at present also, tremendous research and continuous efforts are being made to refine and develop the analytical models for the accurate results. Some representative publications on the subject are available in the work of Beck [1], Ibanez [2], Natke [3], Masri and Werner [4] and Sinha and Kuszta [5]. Very recently, a review paper has also been written for system identification of buildings by Datta et al. [6], where many references may be found related to this topic. To elaborate the survey, few of the recent research that has been done are discussed in the following paragraph.

Loh and Ton [7] studied a system identification approach to detect changes in structural dynamic characteristics on the basis of measurements. They used the recursive instrumental variable method and extended kalman filter algorithm for the identification algorithm. The potential of using neural network to identify the internal forces of typical systems has been investigated by Chassiakos and Masri [8]. A localized identification of many degree of freedom structures is investigated by Zhao et al. [9] and a memory-matrix based identification methodology for structural and mechanical systems are studied by Udwadia and Proskurowski [10]. Chakraverty [11,12] used neural network for the numerical experiment for identification of structural systems where as the identification of stiffness parameters of multistorey frame structure from dynamic data have been studied by Chakraverty [13].

In addition to the above reviewed papers, there exists other research works in the present area of study in the literature. However, the fundamental concepts are similar to those mentioned above.

Recently, Yuan et al [14] have developed an excellent method that estimates mass and stiffness matrices of shear Structure from first two orders of structural mode measurement. A very recent paper by Chakraverty [15] proposed computationally efficient procedures to refine the methods of Yuan et al. [14] to identify the structural parameters from modal test data. The refinement had been obtained by using Holzer criteria [16] along with other numerical techniques [17]. It reveals from the literatures that there exist studies only when either the sensors are in all of the levels or the sensors are placed in first and last level of the structure. Although some have discussed the method in general but in particular, to the best of the present authors' knowledge, it is not investigated at least how many number of sensors are required and at what level for estimation of the full mode shape for a particular order of frequency. So, here a methodology has been proposed from the data which are available by varying the number of sensors and the level of the building where these are installed. As such solutions have also been provided by utilizing a minimum number of data to identify the corresponding structural parameters.

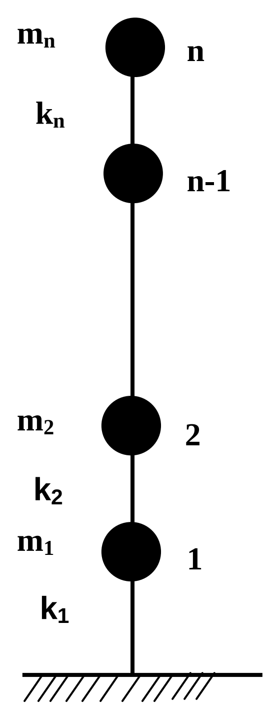

Figure 1. Multistorey structure with n levels.

MATHEMATICAL MODEL

As regards, system identification refers to the branch of numerical analysis, which utilizes the experimental input and output data to develop mathematical models of systems, which finally identify the parameters. In other words, the methodology solves the inverse vibration problems to identify the properties of a structure from the measured data.

For a shear Structure of n levels (Figure 1), the equation of motion subject to ambient vibration may be written as

$$M\begin{Bmatrix}\ddot{y}_1\\ \ddot{y}_2\\ \cdot\\ \cdot\\ \ddot{y}_n\end{Bmatrix}+K\begin{Bmatrix}y_1\\ y_2\\ \cdot\\ \cdot\\ y_n\end{Bmatrix}=\begin{Bmatrix}0\\ 0\\ \cdot\\ \cdot\\ 0\end{Bmatrix} \tag{1}$$

Where M and K are the global mass and stiffness matrices given by

$$M=\begin{bmatrix}m_1 & 0 & \ldots & \ldots & 0\\ 0 & m_2 & 0 & \ldots & 0\\ \ldots & \ldots & \ldots & \ldots & \ldots\\ \ldots & \ldots & 0 & m_{n-1} & 0\\ 0 & \ldots & \ldots & 0 & m_n\end{bmatrix} \textit{and}$$

$$K=\begin{bmatrix}k_1+k_2 & -k_2 & 0 & \ldots & 0\\ -k_2 & k_2+k_3 & -k_3 & \ldots & 0\\ \ldots & \ldots & & & \ldots\\ 0 & \ldots & -k_{n-1} & k_{n-1}+k_n & -k_n\\ 0 & & \ldots & -k_n & k_n\end{bmatrix}$$

where as

$$\{\ddot{y}\}=\{\ddot{y}_1,\quad \ddot{y}_2,\quad \cdots\quad \cdots\quad \ddot{y}_n\}^T \; and \; \{y\}=\{y_1,\quad y_2,\quad \cdots\quad \cdots\quad y_n\}^T$$

are the vectors of acceleration and deflection respectively.

For simple harmonic motion putting the following

$$\{y_1,\quad y_2,\quad \cdots\quad \cdots\quad ,y_n\}^T=\left\{\phi^{(1)},\quad \phi^{(2)},\quad \cdots\quad \cdots\quad ,\phi^{(n)}\right\}^T e^{i\omega t}$$

in Equation (1), these may be written as,

$$\left(K-\lambda_i M\right)\begin{Bmatrix}\phi_i^{(1)}\\ \phi_i^{(2)}\\ \vdots\\ \phi_i^{(n)}\end{Bmatrix}=\begin{Bmatrix}0\\ 0\\ \vdots\\ 0\end{Bmatrix} \tag{2}$$

where $\lambda_i=\omega_i^2$ is the i^{th} eigen value and $\phi_i^{(r)}$, r=1,n designates the i^{th} mode shape or eigen vector.

Determination of Mode Shape from Partial Modal Data

Earlier researchers [14, 15] have given the solution of all components of mode shape for a particular frequency of the structures simply by knowing the first and last level modal data. Here, the authors have derived procedures for getting the full mode shape corresponding to a desired frequency of the structure according as a) the modal data of any two levels are known and (b) modal data for only the first level is known. These three cases are explained below by considering the mass and stiffness values as different for different levels.

Case (a)

Let us first suppose that two sensors are placed at r^{th} and s^{th} levels and 1^{st} to $(r-1)^{th}$, $(r+1)^{th}$ to $(s-1)^{th}$ and $(s+1)^{th}$ to nth level modal components are unknown. Using the known data for r^{th} and s^{th} levels i.e. $\phi_i^{(r)}$ and $\phi_i^{(s)}$ (of the *i*-th mode) in Equation (2), the following matrix equation may be written,

The above equations may be written in matrix form as,

$$
\begin{bmatrix}
P_1 & -k_2 & 0 & 0 & \ldots & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & & & \ldots & & \ldots & & & \ldots & & & \ldots & & \vdots \\
0 & 0 & 0 & \ldots & \ldots & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \ldots & 0 & 0 & \ldots & \ldots & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \ldots & 0 & \ldots & & & \ldots & & & \ldots & & \vdots \\
0 & 0 & 0 & \ldots & 0 & \ldots & & & \ldots & & & \ldots & & \vdots \\
0 & 0 & 0 & \ldots & 0 & \ldots & 0 & 0 & 0 & P_{s+1} & -k_{s+2} & 0 & 0 & 0 \\
0 & 0 & 0 & \ldots & & \ldots & & & \vdots & & & \ldots & & \vdots \\
0 & 0 & 0 & \ldots & & \ldots & 0 & 0 & \vdots & 0 & 0 & 0 & -k_n & k_n - \lambda_i m_n
\end{bmatrix}
\begin{Bmatrix} \phi_i^{(1)} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \phi_i^{(s+1)} \\ \vdots \\ \phi_i^{(n)} \end{Bmatrix}
=
\begin{Bmatrix} 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \\ \vdots \\ k_{s+1}\phi_i^{(s)} \\ \vdots \\ 0 \end{Bmatrix}
\tag{3a}
$$

where $P_t = k_t + k_{(t+1)} - \lambda_i m_t$ for t= 1,2, ….r…s….n.

For a special case of this methodology, if the sensors are placed at 1^{st} and r^{th} level the above matrix equation may be written as

$$
\begin{bmatrix}
P_2 & -k_3 & 0 & 0 & \ldots & \ldots & \ldots & & 0 & 0 \\
0 & & & \vdots & \ldots & \ldots & \ldots & & \vdots & 0 \\
0 & & \ldots & \ldots & & \ldots & & & 0 & 0 \\
0 & & \ldots & & 0 & \ldots & \ldots & & 0 & 0 \\
0 & & \vdots & \ldots & & \ldots & & \vdots & \vdots & 0 \\
0 & & \vdots & \ldots & & \ldots & & \vdots & \vdots & 0 \\
0 & & & \ldots & & \ldots & & \vdots & \vdots & 0 \\
0 & & & \ldots & & \ldots & & \vdots & \vdots & 0 \\
0 & & & \ldots & & \ldots & & \vdots & -k_n & k_n - \lambda_i m_n
\end{bmatrix}
\begin{Bmatrix} \phi_i^{(2)} \\ \vdots \\ \vdots \\ \phi_i^{(j+1)} \\ \vdots \\ \vdots \\ \vdots \\ \phi_i^{(n)} \end{Bmatrix}
=
\begin{Bmatrix} k_2\phi_i^{(1)} \\ 0 \\ \vdots \\ k_{j+1}\phi_i^{(j)} \\ \vdots \\ \vdots \\ 0 \end{Bmatrix}
\tag{3b}
$$

Each of the above matrix Equation (3a) and (3b) may be written in compact form as $[A]\{\phi_u\}=\{Z\}$, where [A] is the square matrix on the left hand side, $\{\phi_u\} = \{\phi_i^{(1)}, \ldots \phi_i^{(s+1)} \ldots\ldots, \phi_i^{(n)}\}^T$ and {Z} is the column vector on the right-hand-side of the above Equations (3a) and (3b).The compact form of the matrices may now be written as

$$\{\phi_u\} = \left(A^T A\right)^{-1}\left(A^T Z\right). \tag{4}$$

Accordingly, Equation (4) gives unknown modal data for the particular order of frequency.

Case (b)

In this case a sensor is supposed to be placed at the first level only, where all other level modal data are unknown. Using the known modal data for first level, the matrix equation may now be given as,

$$\begin{bmatrix} P_2 & -k_3 & 0 & 0 & \ldots & & \ldots & & 0 & \\ 0 & 0 & & & & & & & \vdots & \\ 0 & 0 & \ldots & \ldots & \ldots & \ldots & & & 0 & \\ 0 & 0 & 0 & \vdots & \vdots & \vdots & 0 & \ldots & 0 & \\ 0 & 0 & 0 & 0 & \ldots & \ldots & \ldots & 0 & \vdots & \\ \vdots & & \vdots & \ldots & \vdots & \ldots & & \vdots & \vdots & \\ \vdots & & \vdots & \ldots & \vdots & \ldots & & \vdots & \vdots & \\ \vdots & & \vdots & \ldots & \vdots & \ldots & & \vdots & \vdots & \\ \vdots & & \vdots & \ldots & \vdots & \ldots & & \ldots & -k_n & k_n - \lambda_i m_n \end{bmatrix} \begin{Bmatrix} \phi_i^{(2)} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \phi_i^{(n)} \end{Bmatrix} = \begin{Bmatrix} k_2 \phi_i^{(1)} \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{Bmatrix} \tag{5}$$

Again writing the above equation as $[A]\{\phi_u\}=\{Z\}$, where [A] is the square matrix on the left hand side, $\{\phi_u\} = \{\phi_i^{(2)}, \phi_i^{(3)}, \phi_i^{(4)} \ldots\ldots\ldots, \phi_i^{(n)}\}^T$ and {Z} is the column vector on the right-hand-side of the above Equation (5) respectively gives the modal data for the second to n the level of the multistorey structure.

RESULTS AND DISCUSSIONS

The methodology has been used for various data sets with different number of sensors placed at different levels of the structure. Three problems are solved here, to show the reliability and efficacy of the developed methodologies.

(i)Problem 1

This problem refers to the case (a). Let us consider a ten storey structure. The stiffness and mass values of all the levels are given in Table I. It is supposed that the sensors are placed at first and sixth levels. So, the modal components for the first and sixth levels are available as given in Table II. Equation (3b) is used to evaluate all the components of the mode shapes for 1st and 2nd order frequency parameters. Accordingly Tables III and IV give the evaluated and real values of the mode shapes. These tables show good agreement between the computed and real values.

Table I. The design values of stiffness and mass taken for ten storey structure

Stiffness Values in N/m X 100	k1 /100	k2 /100	k3 /100	k4 /100	k5	k6	k7	k8	k9	k10
	1080	540	540	540	450	405	360	360	360	270
Mass Values in Kg. X 100	m1	m2	m3	m4	m5	m6	m7	m8	m9	m10
	900	360	360	360	270	270	225	216	198	189

Table II. Original Data for ten storey structure (First and Sixth level modal shapes are known)

	First Level		Sixth level	
	Frequency Parameter	Displacement Vector	Frequency Parameter	Displacement Vector
First order	0.0511874	1.000	0.0511874	9.744877
Second order	0.3202796	1.000	0.3202796	1.570133

Table III. Computed values of unmeasured modal components for the ten storey structure when 1st and 6th level modal components are known for 1st order frequency

	$\phi_1^{(2)}$	$\phi_1^{(3)}$	$\phi_1^{(4)}$	$\phi_1^{(5)}$	$\phi_1^{(7)}$	$\phi_1^{(8)}$	$\phi_1^{(9)}$	$\phi_1^{(10)}$
True Value	2.914	4.729	6.383	8.106	11.213	12.323	13.054	13.539
Present method	2.914	4.729	6.383	8.106	11.213	12.323	13.054	13.539

Table IV. Computed values of unmeasured modal components for the ten storey structure when 1st and 6th level modal components are known for 2nd order frequency

	$\phi_2^{(2)}$	$\phi_2^{(3)}$	$\phi_2^{(4)}$	$\phi_2^{(5)}$	$\phi_2^{(7)}$	$\phi_2^{(8)}$	$\phi_2^{(9)}$	$\phi_2^{(10)}$
True Value	2.466	3.405	3.618	2.946	-0.354	-2.208	-3.638	-4.689
Present method	2.466	3.405	3.618	2.946	-0.354	-2.208	-3.638	-4.689

(ii)Problem 2

This problem is an example of case (b). A four storey structure is considered with a sensor placed at first level only. So, the data of the first level are available as given in Table V. All the components of the mode shapes for 1st and 2nd order frequency parameters may be calculated from Equation (5). Tables VI gives the evaluated and real values of all components of the mode shapes for 1st and 2nd order frequencies along with a comparison of data values of reference [15]. These tables show how the computed values match with the real values.

Table V. Original Data for four storey structure (First level modal component is known)

	First Level	
	Frequency Parameter	Displacement Vector
First order	0.2214199	1.00000
Second order	1.3220638	1.00000

Table VI. Computed values of unmeasured modal components for the four storey structure when 1st level modal component is known for 1st and 2nd order frequency

	1st order frequency			2nd order frequency		
	$\phi_1^{(2)}$	$\phi_1^{(3)}$	$\phi_1^{(4)}$	$\phi_2^{(2)}$	$\phi_2^{(3)}$	$\phi_2^{(4)}$
True Value	2.630	3.873	4.544	0.796	-0.108	-0.918
Present Method (Without Holzer)	2.630	3.873	4.544	0.796	-0.108	-0.918
Method of reference [15]	2.630	3.873	4.544	0.796	-0.108	-0.918

(iii)Problem 3

Here the ten storey structure (as discussed in Problem 1) with known modal value of 1st level only is considered which is again an example of case (b). The known modal value is shown in Table VII. Unknown modal components for 1st and 2nd order frequencies are computed from Equation (5) which are incorporated in Tables VIII and IX.

Table VII. Original Data for ten storey structure (First level modal component is known)

	First Level	
	Frequency Parameter	Displacement Vector
First order	0.0511874	1.0000
Second order	0.3202796	1.0000

Table VIII. Computed values of unmeasured modal components for the ten storey structure when 1st level modal component is known for 1st order frequency

	$\phi_1^{(2)}$	$\phi_1^{(3)}$	$\phi_1^{(4)}$	$\phi_1^{(5)}$	$\phi_1^{(6)}$	$\phi_1^{(7)}$	$\phi_1^{(8)}$	$\phi_1^{(9)}$	$\phi_1^{(10)}$
True Value	2.9146	4.7299	6.3837	8.1069	9.7448	11.2135	12.3233	13.0547	13.539
Present method	2.9146	4.7299	6.3837	8.1069	9.7448	11.2135	12.3233	13.0547	13.539

Table IX. Computed values of unmeasured modal components for the ten storey structure when 1st level modal component is known for 2nd order frequency

	$\phi_2^{(2)}$	$\phi_2^{(3)}$	$\phi_2^{(4)}$	$\phi_2^{(5)}$	$\phi_2^{(6)}$	$\phi_2^{(7)}$	$\phi_2^{(8)}$	$\phi_2^{(9)}$	$\phi_2^{(10)}$
True Value	2.466	3.405	3.618	2.946	1.570	-0.354	-2.208	-3.638	-4.689
Present method	2.466	3.405	3.618	2.946	1.570	-0.354	-2.208	-3.638	-4.689

CONCLUSIONS

As already mentioned, the present paper demonstrates the proposed methodology for multistorey shear structures using minimum number of modal information. Result of the example problems show that the method is accurate enough and reliable. The procedures and the methodology are discussed in a concise and simple manner to evaluate the modal components for a particular order of frequency of a multi storey structure if at least modal value of ground level is known.

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