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GENETIC ALGORITHM BASED DAMAGE IDENTIFICATION IN NON-HOMOGENIOUS STRUCTURAL MEMBERS

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ABSTRACT

Non-homogeneous structural members such as beams are very important in various engineering applications and for experimental analysis purposes. A minor damage on any part of the structure reduces the strength of the structure and leads to a major failure. So, the identification of damage is very important and essential at an early stage. In this paper, a new formulation of an objective function for the genetic search optimization procedure along with the residual force method is presented for the identification of macroscopic structural damage in a non-homogeneous beam. The developed model requires experimentally determined data as input and detects the location and extent of the damage in the beam. Here, numerically simulated data using finite element models of structures are used to identify the damage at a reasonable level of accuracy. Damage parameters given theoretically are compared by the present procedure and are found to be in good agreement.

KEYWORDS: Damage factor, Non-homogeneous Material, Genetic Algorithm (GA), Residual Force, Eigen Value, Beam

INTRODUCTION

During the last three decades, vibration based methods have been developed and applied to detect structural damage in the civil, mechanical and aerospace engineering communities ([1] and [2]). These methods are based on the fact that the vibration characteristics of structures (namely frequencies, mode shapes, and modal damping) are functions of the structural physical parameters such as mass, stiffness and damping. Structural damage usually causes a decrease in structural stiffness, which produces changes in the vibration characteristics of the structure. Gawronski and Sawicki [3] employed modal norms to determine damage locations. The residual force concept has received wide attention for application to damage detection and assessment. Residual force provides an objective function to be minimized for achieving the dynamic balance.

Identifying the structural damage with the measured vibration data is an inverse approach in mathematics. The usual damage detection methods minimise an objective function, which is defined in terms of the discrepancies between the vibration data identified by modal testing and those computed from the analytical model ([4] and [5]). However, these conventional optimization methods are gradient based and usually lead to a local minimum only. A global optimization technique is needed to derive a more accurate and reliable solution.

In the last two decades, since first introduced by Holland [6], Genetic Algorithm (GA) has been widely applied to various optimization problems [7]. Many authors have recently taken up this optimization problem using neural networks ([8] and [9]), Genetic Algorithm ([10] and [11]) and neural network with GA [12] by studying the variation of localized damage as a function of modal test data and machine learning. As compared with the traditional optimization and search algorithms, GA search from a population of points in the region of the whole solution space, rather than a single point, and can obtain the global optimum. Moreover, GA has the advantage of easy computer implementation. These properties make GA successful and powerful in the field of structural optimization [13].

As members are frequently operated in extremely thermal and mechanical environment recently, various kinds of new materials have been developed. As one of new materials, functional inhomogeneous materials have received attention, and practical applications of them are anticipated over a multi-scale range from aerospace field to MEMS one. Research and development of functional inhomogeneous materials such as functionally graded materials and composite materials contribute to weight saving and improvement in stiffness or material strength in members. Such weight saving in members often results in considerable decreases in thickness and stiffness, and flexible members cause vibration resulting from time variation of heating, which is called thermally induced vibration. Redecop [14] has studied the free vibration characteristics of the non-homogenous shells. Bhangale and Ganesan [15] have have analysed the buckling and vibration behaviour of functionally graded sandwich beam.When thin-walled members are subject to cyclic thermal and mechanical loads, there exists a lot of chance of failure of the member. There exists many methods to identify damage in a structure in particular to homogeneous beams, plates and shells ([16], and [17]), but to the best of our knowledge method for non-homogeneous structure is scarce. It is due to the fact that the inclusion of the function for the non-homogeneity made the governing equation complex and thereby the damage detection also becomes complex by traditional methods. Accordingly, a powerful and reliable method such as Genetic Algorithm (GA) has been established which may be used intelligently to identify and quantify the damage. Here GA along with residual force vector method has been used for damage identification.

This paper introduces the concept of residual force vector to specify an objective function for an optimization procedure, which is then solved using a Genetic Algorithm. Rao et al. [18] have used this procedure for homogenous cantilever beam, truss structures and portal frames. Panigrahi et al.[18] addressed the problem of damage identification in a cantilever beam of homogenous material only by changing the selection methods in GA. This paper is an extension of Panigrahi et.al. [19] by taking beams of non-homogenous material. Here non-homogeneity parameters are introduced in the governing equation to develop the model. Damage parameters as used corresponds to the reduction in stiffness of an element from which the structure is composed of. GA is employed to determine the values of these parameters by following an iterative process. When the objective function is optimized, values of the parameters indicate the state of the structure. Here, experimental data were simulated numerically by using finite element model of non-homogenous beam and simulations have been done. It is seen that the identified damage factors are in good agreement with the theoretical one. A computer programme using MatLab is employed to find the location and extent of the damage.

RESIDUAL FORCE METHOD

This section describes the construction of dynamics of damaged structures. The governing equation of motion of the dynamics of a multi degree freedom system is given by

$$[M]\{\ddot{X}(t)\} + [K]\{X(t)\} = F(t) \tag{1}$$

where *[M]* and *[K]* are *(n X n)* system mass and stiffness matrices and *X (t)* and *F (t)* are *(n X 1)* physical displacement and applied force vectors.

The j^{th} eigen value equation for ambient vibration associated with equation (1) is

$$[K]\{\phi_j\} - \lambda_j\,[M]\{\phi_j\} = 0 \tag{2}$$

where λ_j and ϕ_j are the j^{th} eigen value and corresponding eigen vector.

In the finite element model of the structure, the global stiffness can be represented as a sum of the expanded element stiffness matrices.

$$[K] = \sum_{i=1}^{m} [k]_i \tag{3}$$

Where k_i represents the expanded stiffness matrix of the i^{th} element and m is the total number of elements.

When damage occurs in a structure, the stiffness matrix of the damaged structure $[K_d]$ can be expressed as a sum of element stiffness matrices multiplied by damage factors associated with each of the m elements α_i (i = 1,2,…m), resulting from the damage.

Then, stiffness matrix of damaged structure may be given by

$$[K_d] = \sum_{i=1}^{m} \alpha_i\,.[k]_i \quad where\ \ \alpha_i \in [0,1]\ and\ \ m = Number\ of\ elements \tag{4}$$

The values of the parameters fall in the range 0 to 1. The value α_i =1 indicates that the element is undamaged and α_i=0 or less than 1 implies completely or partially damaged element respectively.

If it is assumed that the experimental natural frequencies and mode shapes of the damaged structure continue to satisfy the eigen value equation (2), the j^{th} mode of the damaged structure can be written as

$$\left[K_d\right]\{\phi_{jd}\} - \lambda_{jd}\,[M]\{\phi_{jd}\} = 0 \tag{5}$$

where λ_{jd} is the experimentally determined eigen value corresponding to the j^{th} mode shape of the damaged structure. Furthermore as already pointed out, the stiffness matrix is directly affected by the damage and the mass matrix M is assumed to be unaltered.

By substituting equation (4) in equation (5), an expression for residual force vector for j^{th} mode in terms of α_i can be written approximately as

$$R_j = -\lambda_{jd}\,[M]\{\phi_{jd}\} + \sum_{i=1}^{m} \alpha_i\,.[k]_i\ \{\phi_{jd}\} \tag{6}$$

R_j will be zero, only if correct sets of αi are introduced under available damaged modal information λ_{JD} and ϕ_{jd} for a particular mode *j*.

IMPLEMENTATION OF GENETIC ALGORITHM

GA is a search method based on Darwin's theory of evolution and survival of the fittest. Based on the concept of genetics, GA simulates the evolutionary process numerically. Analogous to genes in genetics, GA represents the parameters in a given problem by encoding them in a string. Instead of finding the optimum from a single point in traditional mathematical optimization methods, in GA, a set of points, that is, a population of coded strings, is used to search for the optimal solution. Simple GA consists of three operators: reproduction, crossover, and mutation ([7] and [20]).

To implement GA, it is necessary first to devise a general coding system for the representation of the design variables. Most commonly the design variables are coded by a bit-string. Next step of the procedure is reproduction, which incorporates the concept of natural selection. The fitness of different members of the population must be evaluated before mating to produce the next generation. There are a number of methods of mating pool selection out of which roulette wheel and Tournament selection are mostly used by number of authors for reproduction purpose. In this paper a method known as steady-state selection is selected for reproduction purpose. The main idea of the selection is that bigger part of the chromosome should survive to next generation. After the reproduction phase is over, the population is enriched with better individuals. Crossover operator is applied to the matching pool with a hope that it would create a better string. Following the crossover, the strings are subjected to mutation. The problem presented in the section below for which the search procedure adopted is illustrated by flow chart in Figure 1. The procedure is repeated until the new generation ceases to improve the objective function that shows the occurrence of the convergence.

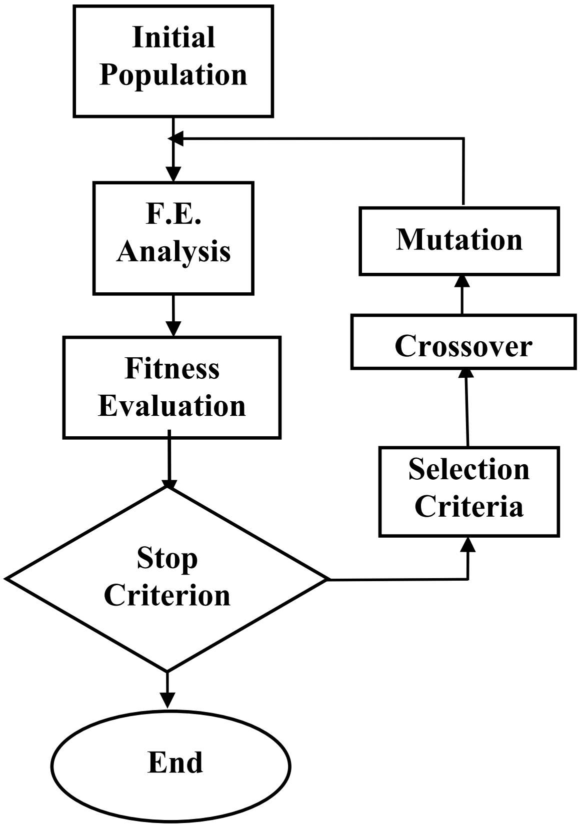

Figure 1. Flow Chart of Genetic Search

OBJECTIVE FUNCTION FOR GA FROM THE RESIDUAL FORCE VECTOR

From equation (6), it is found that the residual force vector is a *(n X n)* matrix where n is number of modes. If $[K_d]$ and $[M]$ are real symmetric matrices it can be shown that the diagonal terms of matrix *[R]* are zero, when a correct set of λ_d and ϕ_d are introduced. Hence the function of damage factors in the present situation is as follows:

$$f(\alpha_1, \alpha_2, ...\alpha_m) = \sqrt{R_{11}^2 + R_{22}^2 + ... + R_{nn}^2} \tag{7}$$

Where *m* is the number of elements and *n* is the number of modes.

Here our problem is to find out first the minimum residual forces. The objective function *V* in the present task is an inverse function defined as below

$$V = C_1 \Big/ \{C_2 + f(\alpha_{1,}\alpha_{2,..........}\alpha_m)\} \quad \text{where } C_1 \text{ and } C_2 \text{ are constants.} \tag{8}$$

The genetic search procedure requires a proper selection of crossover and mutation operators. After some trials, the GA was set up as follows: Population Size -20, crossover probability-0.25 and mutation probability –0.01. Each structural parameter α_i was represented as a 10-bit binary numbers with variable limits 0 to 1.

ILLUSTRATIVE EXAMPLES

Cantilever Beam with Homogenous Material

A cantilever beam with homogenous material (Figure 2) is considered first for the damage detection and extent of the damage using residual force vector method along with genetic algorithm. The beam is simulated numerically with a finite element model taking five elements. Each element is having both translation and rotational degrees of freedom at each nodal point to give a total of twelve. Because the fixed point degree of freedom (dof) has zero rotational and translation movements, the total dof are ten .The properties of the beam chosen are as follows: modulus of Elasticity E = 70.3 GPa; Cross-sectional area A= 1.82 x 10^{-4} m^2; Moment of Inertia I= 1.46 x 10^{-9} m^4; density ρ = 2685 kg/m^3; Total length of the beam l = 0.5 m

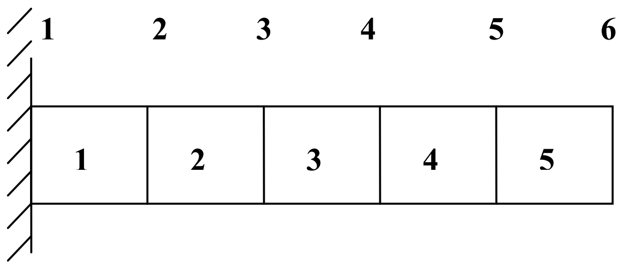

Figure 2. Cantilever Beam Under Consideration

Figure 3 shows the second element of the beam. Q_3 and Q_5 are the transverse displacement and Q_4 and Q_6 are the rotational vectors. There are total four degrees of freedom. For the element as shown l_e=0.1 m and other parameters remain same as of the full beam.

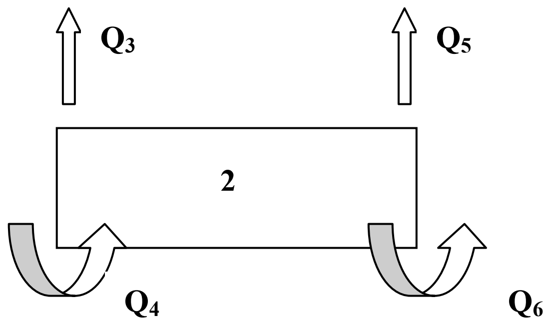

Figure 3. One Element Force Diagram

The element mass and element stiffness matrices for this element may be written as

$$m^e = \frac{\rho A_e l_e}{420}\begin{bmatrix} 156 & 22l_e & 54 & -13l_e \\ & 4l_e^2 & 13l_e & -3l_e^2 \\ & & 156 & -22l_e \\ & & & 4l_e^2 \end{bmatrix} - (9) \quad and \quad k^e = \frac{EI}{l_e^3}\begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ & 4l_e^2 & -6l_e & 2l_e^2 \\ & & 12 & -6l_e \\ & & & 4l_e^2 \end{bmatrix} - (10)$$

Two different situations for this case are considered as (a) beam is in a state of undamaged, (b) the beam having element 2 damaged partially to an extent of 50% and element no. 3 by 30%. From equations 9 and 10, the global mass and stiffness matrices were calculated for undamaged beam. Again by reducing the stiffness of the second element by 50% and 4th element by 30%, the global stiffness and mass matrices were calculated for the damaged structure.

Table 1. Comparison of First Six Natural Frequencies between undamaged and damaged cantilever with homogenous material

Frequency Parameter						
Undamaged beam	203.83	1277.98	3589.46	7090.92	11769.1	19551.8
Damaged beam	179.26	1178.01	3040.38	6425.67	10488.8	17915.3

Now FE analysis is performed to solve the eigen value problem of these two situations and the vibration frequencies are presented in Table 1. It is found as usual that the frequencies in damaged structure in all modes are lower than the undamaged one.

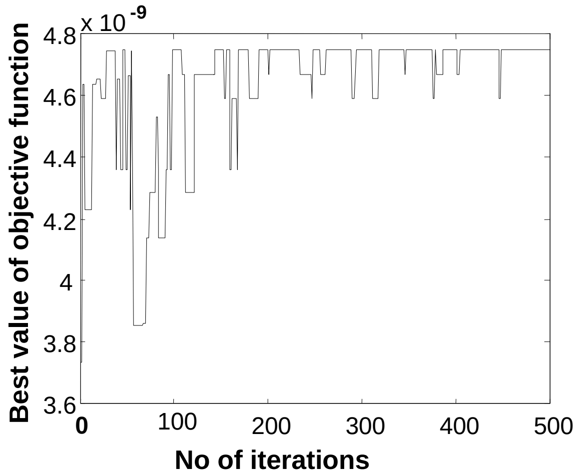

Figure 4. Graph for Maximum Value of Objective Function Verses No. of Iterations

The modal data from Table 1 are employed as input to the model for finding out the values of damage factors from which the location and extent of damage may be identified. Figure 4 shows the best value of the objective function verses the number of iterations. Here, the best value was established at iteration number 25 because when the iteration number was increased there was no improvement in the solution. Accordingly Figure 4 shows this behaviour up to 500 iterations.

Table 2. Results of identified damage factors (α_i) of cantilever beam of homogenous material

Element No.	Undamaged Situation (a)		Damaged Situation (b)	
	Theoretical	Identified	Theoretical	Identified
1	1.0	0.95	1.0	0.95
2	1.0	0.96	0.5	0.47
3	1.0	0.91	1.0	0.93
4	1.0	0.93	0.7	0.67
5	1.0	0.95	1.0	0.98

From Table 2 it reveals that in both the damaged and undamaged situations the theoretical and GA identification approach are in good agreement. Percentages of error for both the cases are incorporated in this table.

Cantilever Beam with Non-homogenous Material

Here the same cantilever beam (Figure 2) with non-homogenous material is considered. The modulus of elasticity varies along with the length of the beam. The modulus of elasticity at any distance x from free end is given by $E=E_0\,(1+r)$ where E_0 = Modulus of elasticity at the free end and r = x / l and other parameters same as previous example. Figure 6 shows the third element of the tapered beam. Q_5 and Q_7 are the transverse displacement and Q_6 and Q_8 are the rotational vectors. There are total four degrees of freedom. For the element as shown l_e=0.1 m and other parameters remain same as of the full beam.

Putting Eq. 12 and 13 in Eq. 9 and 10, m^e and k^e will both become the functions of the taper parameters. By putting the appropriate thickness parameter, the values can be computed.

Four different situations with r = 0.006 for this case are considered as (a) beam is in a state of undamaged, (b) the beam having element 3 damaged partially to an extent of

40% (c) the beam having element 2 damaged partially to an extent of 50% and element number 4 to an extent of 30%.

From equations 9 and 10, the global stiffness and mass matrices were calculated for undamaged beam putting original values of stiffness parameters for the case (a). For the case (b) the global matrices were calculated by reducing the stiffness value of the 3^{rd} element by 40% and for the case (c) by reducing the stiffness value of the 2^{nd} element by 50% and 4^{th} element by 30%.

Figures 5 and 6 show the best value of the objective function verses the number of iterations for case (b) and case (c) respectively.

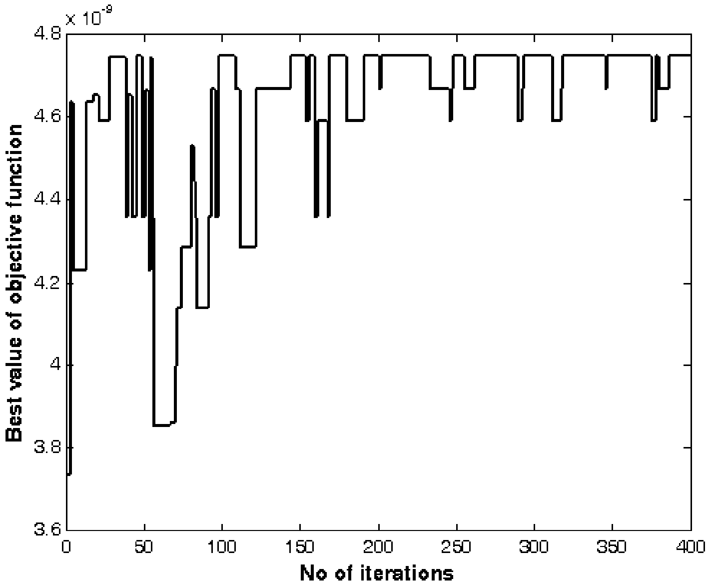

Figure 5. Graph for Maximum Value of Objective Function Verses No. of Iterations

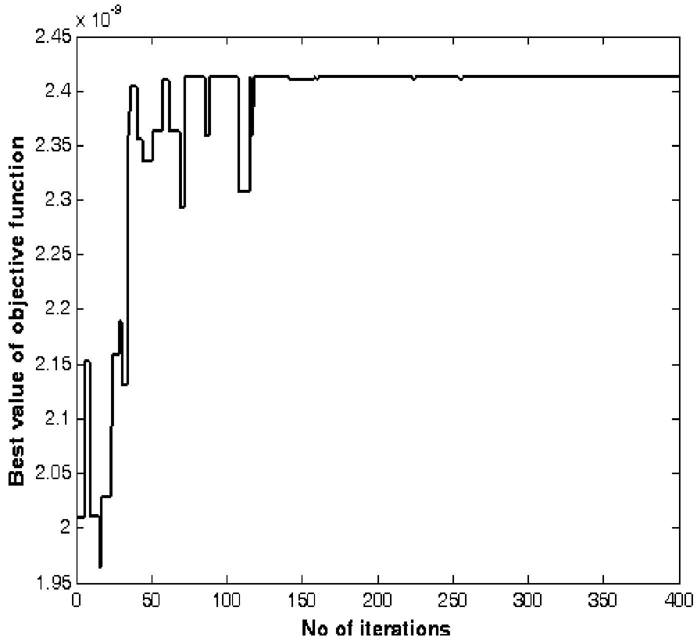

Figure 6. Graph for Maximum Value of Objective Function Verses No. of Iterations

As discussed previously, it may be seen from figures 7 and 8 that the best value is achieved at 28 and 32 number iterations respectively. Again it is worth mentioning that there is no further development of the best value after these iterations.

Table 3. Results of identified damage factors (αi) of non-homogenous cantilever beam

Ele-ment No	Undamaged Situation Case (a)		Damaged Situation Case (b)		Damaged Situation Case (c)	
	Theoretical	Identified	Theoretical	Identified	Theoretical	Identified
1	1.0	0.96	1.0	0.95	1.0	0.97
2	1.0	0.95	1.0	0.94	0.5	0.48
3	1.0	0.97	0.6	0.56	1.0	0.97
4	1.0	0.89	1.0	0.92	0.7	0.64
5	1.0	0.93	1.0	0.98	1.0	0.93

From Table 3 it reveals that in both damaged and undamaged situations the theoretical and GA identification approach are for the present problem in good agreement.

SPECIAL CASE

Here, the homogenous structure equation may be obtained simply by putting r=0 in non-homogenous structure equation i.e. making it to a homogenous beam. The case (c) in the second example is solved by putting r=0 and compared with the case (b) of the first example. They are found to be in good agreement, which shows the reliability of the model for the non-homogenous beam.

CONCLUSION

A procedure has been presented for the simultaneous location and quantification of the damage in engineering structures with non-homogenous materials. Genetic algorithms have been employed for which the optimization function has been formulated in terms of modified residual force vectors. The damage factors identified for the beam problem, which are obtained by using GA for optimization purpose, show excellent agreement with those chosen for the mechanical simulation of these damaged structures.

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REFERENCES

[1] Cawley P, and Adams R.D. (1979),“The location of defects in structures from measurement of natural frequencies”, J. Strain.Anal.Vol.14, No 2, PP 49-47.

[2] Stubbs N, Osegueda R. (1990), “Global damage detection in solids: experimental verification”, Int. J. Anal. Exp. Modal Anal. Vol.5, No 2, PP 81-97.

[3] Gawronski W, Sawicki J.T. (2000), “Structural damage detection using modal norms”, J Sound Vibration, Vol.229, PP 194-198.

[4] Hajela P, Soeiro F.J. (1990), “Structural damage detection based on static and modal analysis”. AIAA J. Vol. 28, No 4, PP 1110-1115

[5] Chakraverty S. Singh V.P. and Sharma R.K.(2006), “Regression based weight generation algorithm in neural network for estimation of frequencies of vibrating plates”, Computer methods Appl. Mech. Engg. Vol.195, PP 4194-4202.

[6] Holland J.H. (1975),“Adaption in natural and artificial systems”, University of Miichigan, Ann Arbor, Michigan.USA.

[7] Goldberg D.E. (1989),“Genetic algorithms in search, optimization and machine learning”, Addsion-Weesley Longman, Reading, Mass.

[8] Tsou P, Shen M.H.H. (1994), “Structural damage detection and identification using neural networks”, AIAA J Vol. 32, PP 176-183.

[9] Barai S.V., Pandey P.C. (1995), “Vibration signature analysis using artificial neural networks”, J Comp Civil Engg (Trans ASCE), Vol. 9, PP 259-265.

[10] Mares C, Surace C. (1996), “An application of genetic algorithm to identify damage in elastic structures”. J. Sound Vib. Vol. 195, PP 195-215.

[11] Luh G.C. and Wu C. Y.(1999), “Nonlinear system identification using genetic algorithms”, J Sypt Control, Proc Imech E. Vol. 213, PP 105-112.

[12] Marawala T, Chakraverty S. (2006), “Fault classification in structures with incomplete measured data using autoassociative neural networks and genetic algorithm”,Current Science, Vol. 90, No 4, PP 542-548.

[13] Rajasekaran S, Pai G.A.V.(2003),“Neural networks, fuzzy logic, and genetic algorithms, synhesis and applications”, Prentice-Hall of India Pvt. Ltd., New Delhi.

[14] Redecap D., (2006), “Three dimensional free vibration analysis of in-homogenous thick orthotropic shells of revolution using differential quadrature”, J. of Sound and Vibration, Vol.291, PP-1029-1041.

[15] Bhangale R.K. and Ganesan N.,”Thermoelstic buckling and vibration behavior of a functionally graded sandwich beam with constrained viscoelastic core”,J. of Sound and Vibration,

[16] Subramanian G, Raman A. (1996), “Isospectral Systems for Tapered Beams”. Journal of Sound and Vib. Vol. 198, No 3, PP 257-266.

[17] Ashour A. S. (2001), “A semi-analytical solution of the flexural vibration of orthotropic plates of variable thickness”, Journal of Sound and Vibration, Vol. 240, No 3, PP 431-445.

[18] Rao M. Anand, Srinivas J. and Murthy B.S.N. (2004), “Damage detection in vibrating bodies using genetic algorithms”, Computers and Structures, Vol. 82, PP 963-968.

[19] Panigrahi S K, Chakraverty S. and Mishra B K. (March, 2007, “Identification of damage in structural members using genetic algorithm”, National Conference on Emerging Technology and Developments in Civil Engg. ETDCE-07, 22-23, Govt. Engg. College, Amaravati, PP-V71- 78.

[20] Michalewicz Z. (1994),“Genetic algorithms + data structure = evolution programs”, Spring-Verlag, Second, Extended edition.