BUILDING DIGEST

CENTRAL BUILDING RESEARCH INSTITUTE INDIA

FOLDED PLATE ROOFS

Introduction

Folded-plates, also known as 'Hipped-plates' or prismatic shells are an economical means of roofing large column-free areas required for factory buildings, storage structures, assembly-halls and schools. Though quite popular in the U.S.A. their introduction to India is rather recent. The trough-type of folded-plate appears to have been employed for the first time in roofing the Museum of the Central Building Research Institute, Roorkee. This was followed by another major application of northlight folded plates to roof the Workshop of the Central Scientific Instruments Organisation, now under construction at Chandigarh. These two projects have attracted considerable attention among engineers and architects and many enquiries are being received on the design and construction of this form of construction. This digest attempts to answer some of the questions often asked.

Different Shapes

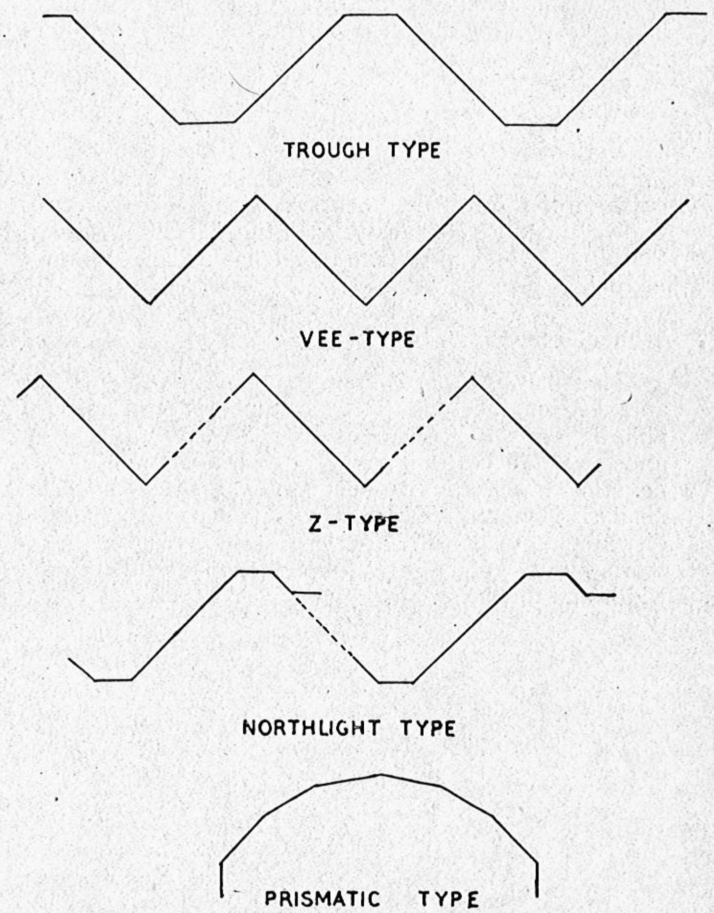
A proper choice of shape leads to economy in cost, functional suitability and aesthetic appeal. Hence, a designer should consider these factors in choosing the appropriate shape.

Certain shapes are structurally more efficient than others. With the increase in height of the cross section, structural depth increases which in turn reduces the longitudinal stresses. At the same time, if the width of the plate also increases, the transverse moments increase. Since the major quantity of steel in folded plates is meant to cater for longitudinal stresses and transverse moments, the designer should aim at keeping the total steel required for both these stress resultants to the minimum.

Folded plates, in general, have good light reflecting geometry. The shape chosen should help in distributing the light uniformly. Thus, the north-light folded plate roof ensures a very even distribution of lighting and is ideally suited for factory buildings.

The shape of the cross-section influences ease in construction. Sharp edged V-shapes involve congestion of steel and hence proper compaction of concrete poses a problem. The trapezoidal shape, with relatively small top and bottom plates, is thus an improvement over the V-shape.

Prismatic shells (fig. 1) have greater structural depth and resemble cylindrical shells. Unsymmetrical sections such as shown in figure 1 are suitable for north-light roofing.



DIFFERENT SHAPES

FIG. NO. 1

Choice of Dimensions

(i) **Height**—The structural depth of folded plates of the prismatic shell type may be assumed as between 1/8 to 1/12 of span. For any other shape, the depth may be taken as 1/12 to 1/15 of span for preliminary designs.

(ii) **Thickness**—The thickness of folded plates generally varies between 3″ to 5″. Greater thickness is required to resist larger transverse moments. Thickness less than 3″ is not desirable. Sometimes, the thickness of top and bottom plates of trapezoidal sections is increased to resist large compressive and tensile stresses.

(iii) **Inclination of the plates**—The plates should not be inclined at more than 42° to the horizontal to avoid the need for back forms while concreting. At the same time, larger the inclination, greater is the structural depth and hence the inclination should not be unduly reduced. It is advisable to keep the inclination between 35° to 42°.

(iv) **Thickness of Diaphragm**—Thickness of the diaphragm is generally 8″ to 12″, depending on the span and depth of the folded plate. The width chosen should be adequate to accommodate the steel provided in the diaphragm. As the height of the diaphragm increases, pouring of concrete in narrow section becomes a problem.

Analysis

Various methods are available for the analysis of folded plates. The basic principles of analysis are common to most of the theories. Some of the well-known methods of analysis are those due to Whitney, Simpson, Winter and Pei and Yitzhaki. The Whitney method is presented below :

Whitney's theory :*

The analysis is based upon the continuity of stress and deformation along the common edge of adjacent plates. Thus, two equations are available at each joint and the number of unknowns at each joint is also two i.e. the transverse moment and longitudinal shear. Analysis is greatly simplified by expressing the uniformly distributed load in Fourier series. This, however, holds good only for folded plates of constant width and thickness.

Notations : (fig. 2)

w_k — width of the plate k between longitudinal edges.
b_k — horizontal projection of width of the plate k
t_k — thickness of the plate k
γ_k — angle between the plates k and k+1 measured in clock wise direction from the plate k
q_k — load intensity
R_k — total transverse plate load on the plate k
T_k — total longitudinal edge shear force at the joint k
$M_{o,k}$ — plate moment caused by transverse load on the plate k
m_k — transverse moment at the plate edge k
A_k — cross-sectional area of the plate k
Z_k — section modulus of the plate k for 'plate action'
I_k — moment of inertia of the plate k for 'plate action'
J_k — moment of inertia of the plate k for 'slab action'
Δ_k — deflection of the plate k for 'plate action'
θ_k — angle change at the joint k due to applied load
ψ_k — angle change at the joint k due to transverse moment
ν_k — angle change due to plate deflection at the joint k
L — length of plate between transverse supports

The important steps of this method are as follows :

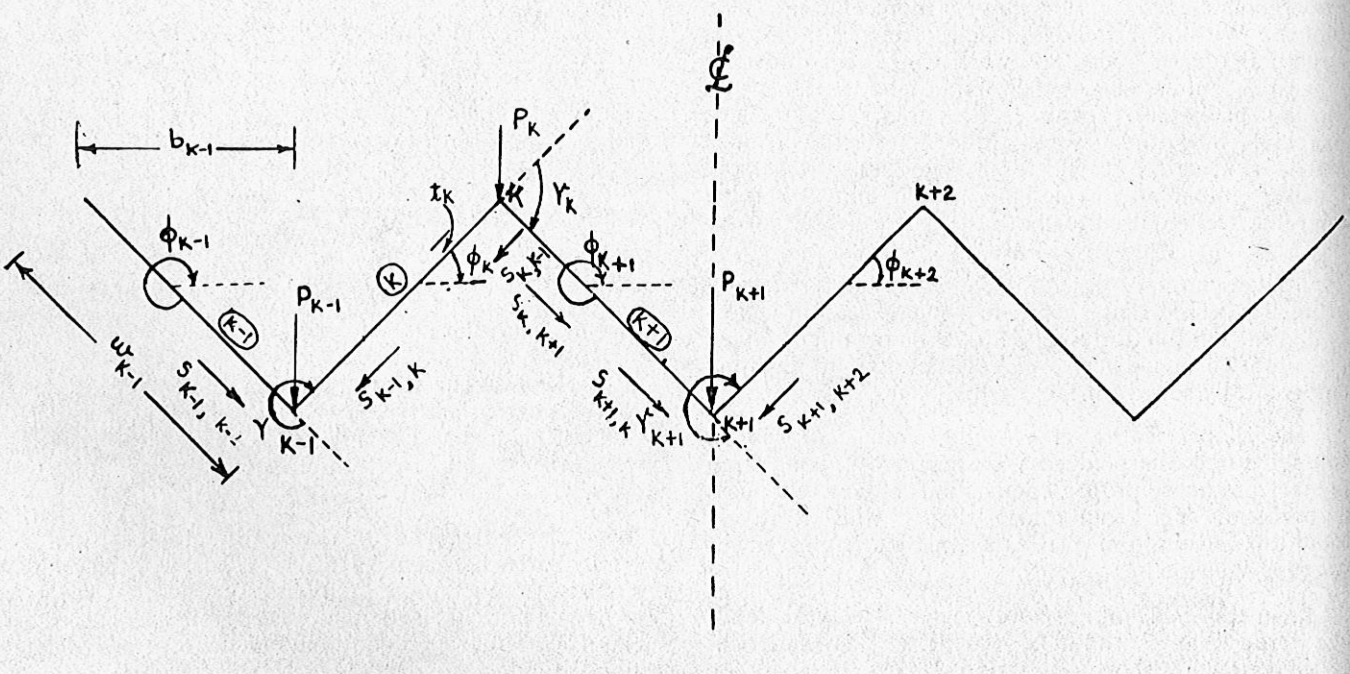


FIG. NO. 2.

* This theory has been published in the proceedings of the Structural Division of A.S.C.E. Oct. 1959

(i) Each plate is assumed to be a simply supported slab spanning between the adjacent plates and the joint loads are then calculated and replaced by their components in the planes of the plates. These form the initial plate loads.

(ii) The continuity of the plates over the joints induces transverse moments which cause additional joint loads. These additional loads are calculated in terms of the unknown transverse moments.

(iii) Under the action of plate loads calculated in (i) and (ii) each plate bends in its own plane between the diaphragms. The bending moments and the longitudinal stresses induced by them are calculated. Continuity at the joint demands that the longitudinal stress should be the same in both the plates at the common edge. This can be realised by introducing edge shears at each joint.

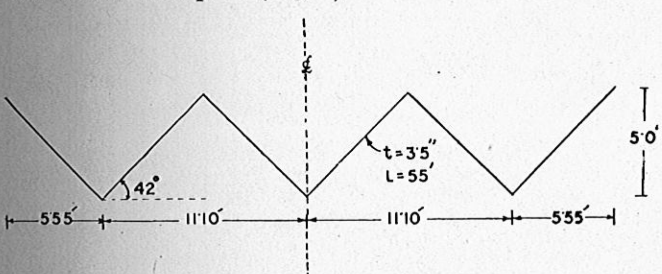
(iv) The individual plate deflections caused by the transverse loads and the edge shears are calculated in terms of the applied loads and the unknown transverse moments.

(v) At each joint, the condition of zero angle change is written down in terms of the applied loads, transverse joint moments and plate deflections.

(vi) By solving for the unknown transverse moments in (v), all other stress resultants are obtained.

The example that follows illustrates the procedure just outlined. The work involved has been organized in 9 tables. Intermediate calculations between the tables have been omitted.

Numerical Example : (Fig. 3)



NUMERICAL EXAMPLE

FIG. NO. 3

Thickness of the plate	=	3.5 inches
Span of the folded plate	=	55 ft.
Live load on the roof	=	10 lbs/sq. ft. of surface area
Self weight of the roof	=	44 lbs/sq. ft.

Because of symmetry, only half of the structure is considered.

TABLE I

Geometry

Plate	Plate width (feet) (w_k)	Horizontal Projection (feet) (b_k)	Inclination to horizontal (ϕ_k)	Angle between the Plate & the next plate (γ_k)
1	7.473	5.553	318°	276°
2	7.473	5.553	42°	84°
3	7.473	5.553	318°	276°

TABLE II

Sectional Properties

$$A_k = t_k \cdot \omega_k$$
$$Z_k = \tfrac{1}{6}\, t_k \cdot \omega_k^{2}$$
$$I_k = \tfrac{1}{12}\, t_k \cdot \omega_k^{3}$$

Plate	A_k (ft^2)	Z_k (ft^3)	I_k (ft^4)
1.	2.1796	2.7147	10.1435
2.	2.1796	2.7147	10.1435
3.	2.1796	2.7147	10.1435

TABLE III

Joint Loads

$$P_k = \frac{4}{\pi}\left[\tfrac{1}{2}\, q_k\omega_k + \tfrac{1}{2} q_{k+1}\omega_{k+1}\right] \text{Sin } \frac{\pi x}{L}$$

$$\triangle P_k = \left[\frac{m_k - m_{k+1}}{b_{k+1}} - \frac{m_{k-1} - m_k}{b_k}\right] \text{Sin } \frac{\pi x}{L}$$

$$\text{and } \triangle P_1 = \left[\frac{m_1 - m_2}{b_2}\right] \text{Sin } \frac{\pi x}{L}$$

(at the joint No. 1, entire load of cantilevered plate is considered unlike at other joints)

Joint	P_k (lbs.)	$\triangle P_k$
1.	773.64	$0.18m_1 - .18m_2$
2.	513.604	$-.18m_1 + .36m_2 - .18m_3$
3.	513.604	$-.36m_2 + .36m_3$

TABLE IV

Plate Loads

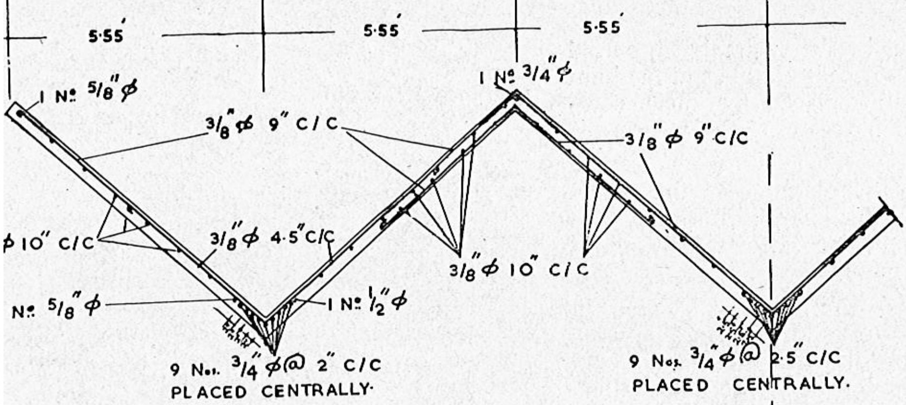
$$S_{k,k-1} = P_k \frac{\cos \phi_{k+1}}{\text{Sin } \gamma_k} \qquad \triangle S_{k,k-1} = \triangle P_k \frac{\cos \phi_{k+1}}{\text{Sin } \gamma_k}$$

$$S_{k,k+1} = P_k \frac{\text{Cos } \phi_k}{\text{Sin } \gamma_k} \qquad \triangle S_{k,k+1} = \triangle P_k \frac{\text{Cos } \phi_k}{\text{Sin } \gamma_k}$$

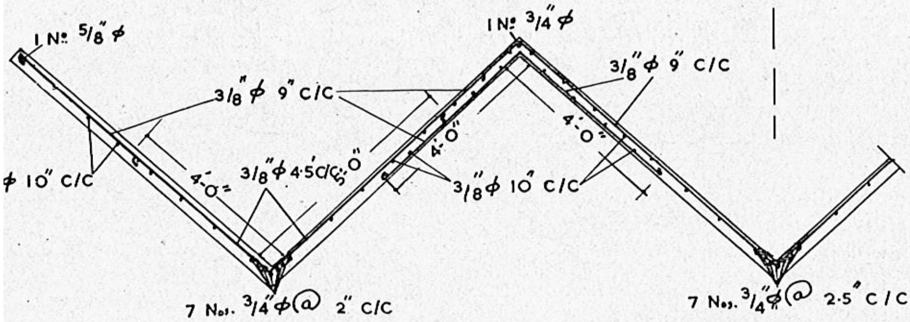
$$\text{and } R_{k,k-1} = \left[S + \frac{4}{\pi} \triangle S\right]_{k,k-1}$$

(Plate load acting towards inferior side is considered positive)

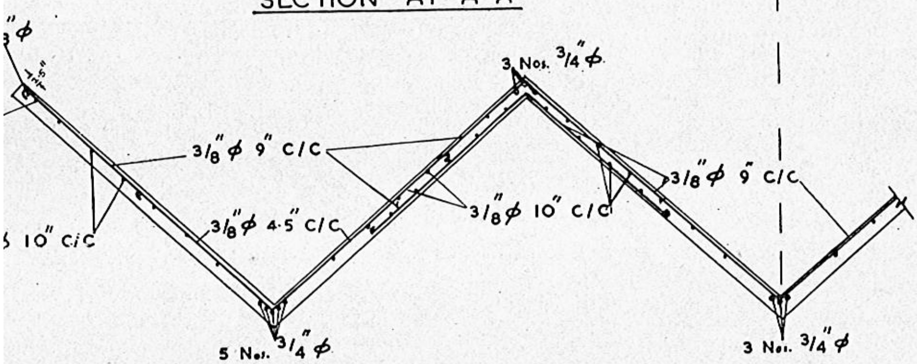
Load Direction	S (lbs)	$\triangle$ S	R
1—0	−578.10	$-(.1345m_1 - .1345m_2)$	$-(578.10 + .1712m_1 - .1712m_2)$
1—2	+578.10	$+(.1345m_1 - .1345m_2)$	$+(578.10 + .1712m_1 - .1712m_2)$
2—1	+384.50	$+(.1345m_1 + .269m_2 - .1345m_3)$	$+(384.5 - .1712m_1 + .3424m_2 - .1712m_3)$
2—3	−384.50	$-(.1345m_1 + .2690m_2 - .1345m_3)$	$-(384.5 - .1712m_1 + .3424m_2 - .1712m_3)$
3—2	−384.50	$-(-.2690m_2 + .2690m_3)$	$-(384.5 - .3424m_2 + .3424m_3)$



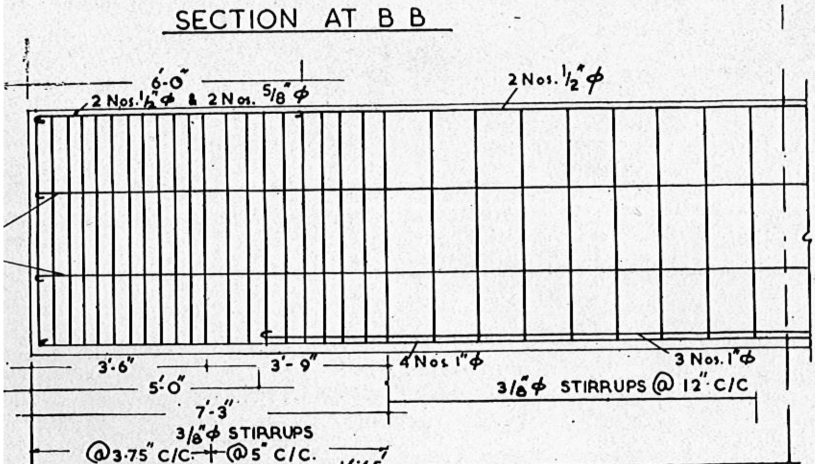
SECTION AT MID SPAN



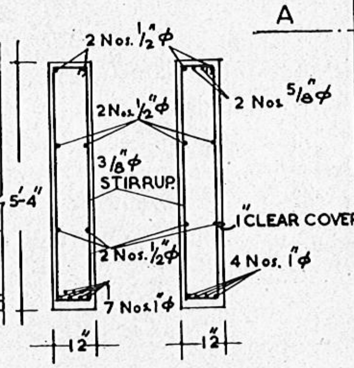
SECTION AT A A



SECTION AT B B



SECTIONAL ELEVATION OF DIAPHRAGM



AT MID SPAN AT SUPPORT

CROSS SECTIONS

RS a, b, c, & d REPRESENT 3/4", 5/8", 1/2" & 3/8" DIA. RESPECTIVELY AND
E CURTAILED AS SHOWN IN THE PLAN.

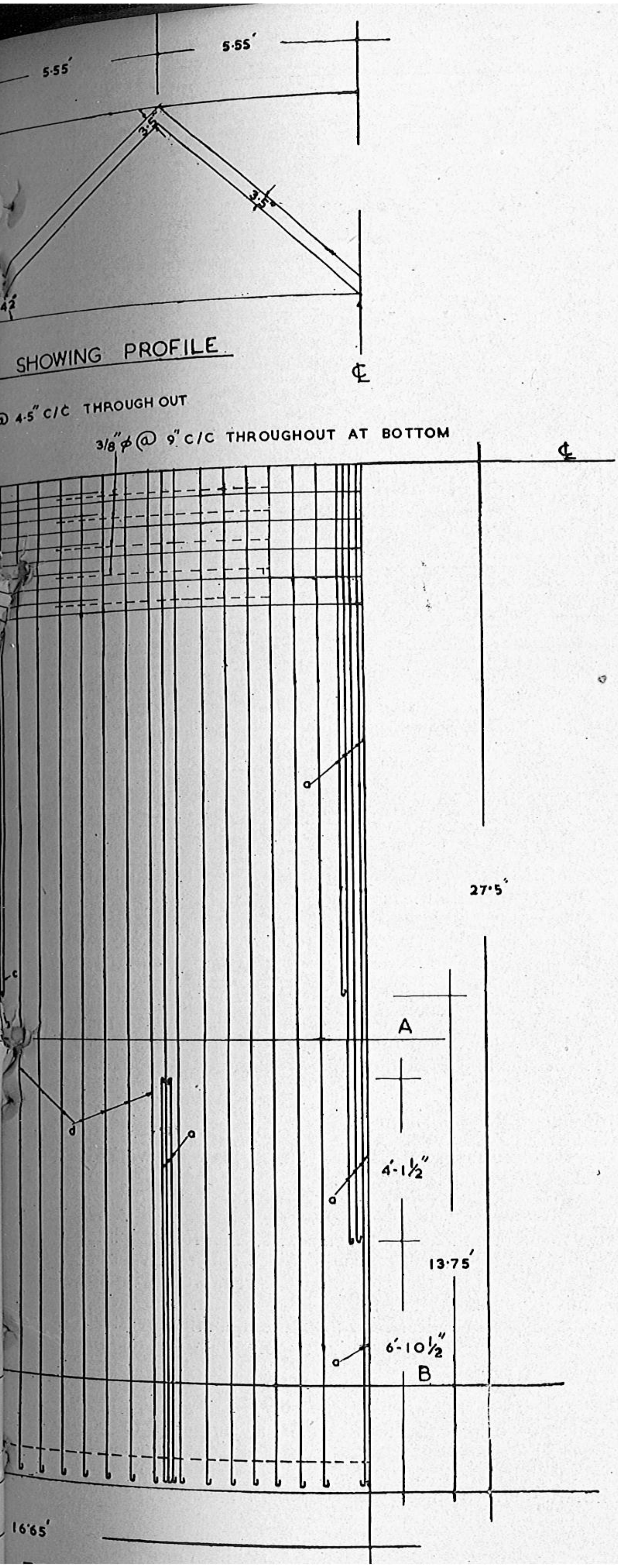


TABLE V

Plate Moments

$$M_{0,\,k} = R_k \frac{L^2}{\pi^2}$$

$$R_k = R_{k,\,k-1} + R_{k-1,\,k}$$

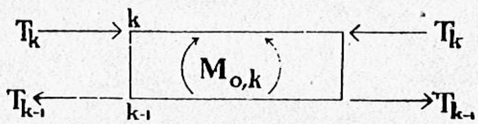
Plate	Plate load (R_k)	Plate moment ($M_{o,\,k}$)
1.	$-769.92+.1712m_2$	$-235980.48+52\,473m_2$
2.	$+962.6+.1712m_2$ $-.1712m_3$	$+295036.9+52.473m_2$ $-52.473m_3$
3.	$-577.18-.1712m_3$	$176905.67-52.473m_3$

TABLE VI

Edge Shear Forces

$$\frac{T_{k-1}}{A_k} + 2T_k\left(\frac{1}{A_k} + \frac{1}{A_{k+1}}\right) + \frac{T_{k+1}}{A_{k+1}} = -\tfrac{1}{2}\left[\frac{M_{o,\,k}}{Z_k} + \frac{M_{o,\,k+1}}{Z_{k+1}}\right]$$

Positive directions of longitudinal shears and moments are as shown here :



The above equation is written at every internal edge On solving such equations and substituting for $M^s_{o,\,k}$ in terms of m^s_k, values of T^s_k are obtained as shown below

Edge	Shear force (T_k)
1.	$-3147-9.827m_2+2.804m_3$
2.	$-11060-2.809m_2+9.837m_3$
3.	0

TABLE VII

Plate deflections

$$\triangle_k = \frac{1}{EI_k}\left(\frac{L}{\pi}\right)^4\left[R_k + (T_{k-1}+T_k)\left(\frac{\pi}{L}\right)^2\frac{w_k}{2}\right]\sin\frac{\pi x}{L}$$

Substituting R^s_k and T^s_k from tables V and VI, $\triangle^s_k$ are obtained

Plate	Deflection ($\triangle_k$)
1.	$\frac{10^3}{E}$ $[-7491.5+0.475m_2+.3168m_3]$
2.	$\frac{10^3}{E}$ $[+7318-.159m_2-.159m_3]$
3.	$\frac{10^3}{E}$ $[-6600-.3174m_2-.477m_3]$

TABLE VIII

Change of angle

I Angle change due to load

$$\theta_k = \frac{4}{\pi}\left[\frac{q_k \omega^2_k b_k}{2\,E\,t^3_k} + \frac{q_{k+1}{}^2 \omega_{k+1}\, b_{k+1}}{2\,E\,t^3_{k+1}}\right]$$

II Angle change due to Transverse moment

$$\psi_k = \frac{4}{\pi}\left[\frac{\omega_k(2m_k+m_{k-1})}{6\,E\,J_k} + \frac{\omega_{k+1}(2m_k+m_{k+1})}{6\,E\,J_{k+1}}\right]$$

III Angle change due to plate deflections

$$\nu_k = \frac{1}{w_k}\left[\triangle_k(\text{Cot}\,\gamma_k + \text{Cot}\,\gamma_{k-1}) - \frac{\triangle_{k+1}}{\text{Sin}\,\gamma_k} - \frac{\triangle_{k-1}}{\text{Sin}\,\gamma_{k-1}}\right]$$

$$-\frac{1}{\omega_{k+1}}\left[\triangle_{k+1}(\text{Cot}\,\gamma_{k+1} + \text{Cot}\,\gamma_k) - \frac{\triangle_{k+2}}{\text{Sin}\,\gamma_{k+1}} - \frac{\triangle_k}{\text{Sin}\,\gamma_k}\right]$$

Joint	Angle change due to load	Angle change due to transverse moments	Angle change due to deflections of plates
2	$\frac{10^3}{E} \times 859.4$	$\frac{4}{\pi} \times \frac{10^3}{E}$ [.6026m_1 +2.4104m_2 +0.6026m_3]	$\frac{10^3}{E}$ [—23.3+ .08531m_2+ .02124m_3]
3	$\frac{10^3}{E} \times 859.4$	$\frac{4}{\pi} \times \frac{10^3}{E}$ [1.2052m_2 +2.4104m_3]	$\frac{10^3}{E}$ [—193.4 +.04266m_2 +.17114m_3]

But $\quad -\theta_k + \psi_k + \nu_k = 0$

Solving equations formed at joints where the change in angle is zero, values of m_2 and m_3 are obtained. m_1 is a statically determinate quantity and is equal to

$$54 \times 7.473 \times \frac{5.533}{2} = 1120.427 \text{ ft. lbs.}$$

Substituting values of m_2 and m_3 in Table No. V and VI, plate moments and edge shear forces are obtained. Knowing edge shear forces and plate moments, longitudinal stresses can be obtained.

TABLE IX A

Solution

Joint	Transverse moments (lb. ft.)	Longitudinal shear (lb.)	Longitudinal stress (lb./ft.^{2})
0	0	0	—89754.3967
1	+1120.427	—1278.0613	+90608.0022
2	—85.6085	—7214.2722	—85165.6793
3	+366.4993	0	+78869.7508

Longitudinal stresses shown above are the mean stresses at each edge. They may be multiplied by a factor $\pi^3/32$ to obtain longitudinal stresses due to uniformly distributed loading.

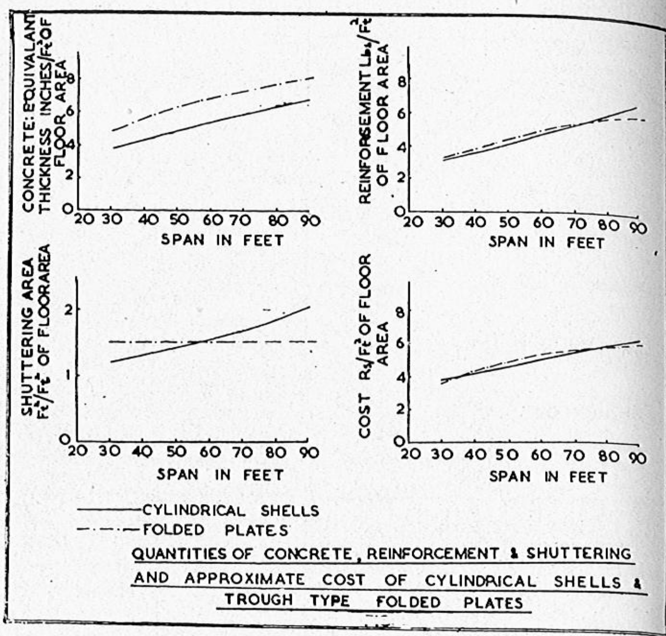


Fig. 5

TABLE IX B

Plate	Plate moments (lb. ft.)	Plate loads (lb./ft.)
1	—240472.6148	—784.5762
2	+271313.4474	+885.1992
3	—196136.9878	—639.9246

Static checks should be applied to the longitudinal stresses worked out in table IX such that the total compression on the section is equal to the total tension and the internal moment of resistance of the cross-section is equal to the external bending moment at that section.

As meagre information is available about the buckling aspect of folded plates, it is advisable to restrict the longitudinal compressive stresses to 750 psi.

Design of Reinforcement

Reinforcement in folded plates may be worked out in one of the three ways :

(i) *Design based on 'Homogeneous Section'* : As per assumption made in the analysis, the folded plate is assumed to be homogeneous. Since concrete cannot take any tension, the tension zone is reinforced with an equivalent quantity of steel. This procedure, however, leads to uneconomical design as the quantity of steel obtained in this way is much more than that due to other methods.

(ii) *Design based on 'Individual plates'* : Since the concrete below the neutral axis is supposed to be cracked, a more rational approach of providing steel is to consider the individual plate subjected to the known plate moment and edge shear forces. Care should be taken to see that the stresses attained in two plates at their common joint are the same, thus ensuring compatibility of stresses at the joint. Reinforcement based on this principle is shown in Fig. 4.

(iii) *Design based on 'Beam Behaviour'* : Sometimes the longitudinal stress-distribution in the folded plate may resemble that in a beam. In such cases, it is logical to match the internal resisting moment obtained by considering the entire cross-section to the external bending moment and provide steel on that basis.

Design of Diaphragm.

Simple rectangular diaphragms take the place of complicated transverses required for shells. The diaphragm is designed as a simply supported beam subjected to inclined plate loads obtained in Table IX B. The reinforcement provided is shown in Fig. 4

Constructional aspects

Shuttering : Shuttering required is relatively simpler, as it involves only straight planks. Timber boxes, conforming to the shape of the folded plate, supported on ballies can be used as shuttering. However, it is advantageous to use movable type of shuttering. It may consist of collapsible type of formwork resting on a trusswork which moves parallel to the diaphragms. The collapsible unit of the formwork may be either in plywood or in black sheet both being stiffened by timber battens. The plywood shuttering used in the construction of the Museum building of Central Building Research Institute had six reuses. The use of black sheets for formwork is justified only when the number of reuses is large. The shuttering units can be used a large number of times, if the structure is decentered earlier. The shuttering of the Museum Building of the Central Building Research Institute was stripped at the end of 7 days only. This was mainly due to the greater rigidity of folded plates as compared to cylindrical shells. However, care should be taken to ensure that the strength attained by the work cubes is not less than 80 percent of the 28 day strength at the time of demoulding.

Compaction of concrete :

It is very essential to ensure that the concrete is properly compacted and no honey-combing occurs at the soffit. The concrete mix should be sufficiently stiff, if a vibrator is used. Slump of 1½" to 2½" is generally sufficient for slopes up to 45°. Different vibrators have been employed in different ways. Needle vibrator is generally found suitable for the job. But its operation requires some skill to ensure that the concrete does not flow down. Sometimes the needle vibrator is held against the reinforcement to transfer uniform vibrations to the concrete. Form vibrators are used, if the centering and shuttering are strong enough to take up the vibrations. As the vibration of the needle vibrator is concentrated at a particular point, it is sometimes found convenient to use the Kango Hammer held against a plank resting on the concrete which is being compacted.

Water Proofing and Thermal Insulation

As the valleys are closed from both ends by the diaphragms, water is likely to stagnate in it. To facilitate drainage, a slope of 1 in 100 may be given to the roof itself. Bituminous felts with or without aluminium paint, Aluminium foil with suitable adhesive and separated from concrete surface by a bituminous layer are suitable waterproofing treatments. Thermocole or foam concrete in combination with aluminium foil or bituminous water proofing felt would offer thermal insulation in addition to waterproofing.

Economics

The use of relatively simple shuttering of straight planks brings down the shuttering cost. But as the folded plates consume slightly more cement and steel as compared to cylindrical shells, the increased cost on this account is compensated by the lower shuttering cost. The economical range of spans for folded plate roofs can be decided from Fig. 5 which compares the quantities of concrete, steel, shuttering and approximate cost for folded plates and cylindrical shells of various spans.

There is a demand for short notes summarising available information on selected building topics for the use of Engineers and Architects in India. To meet the need this Institute is bringing out a series of Building Digests from time to time and the present one is the twenty first in the series.

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