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Failure criterion of vertical shear key joints in prefabricated wall panels

S. C. Chakrabarti, N. N. Bhise and K. N. Sharma

Vertical shear key joints in prefabricated wall panel assemblies play an important role in the design of prefabricated shear wall systems. To determine the load carrying capacity and other shear deformability characteristics of the joints, a test programme was undertaken at the Central Building Research Institute, Roorkee. In the paper, an attempt has been made to predict the failure load of vertical joints by the modified Coulomb's failure criterion and to compare the predicted values with the test results. It was observed that the load carrying capacity of reinforced, and within certain limitations, unreinforced vertical shear key joints can be predicted by Coulomb's modified failure criterion.

In case of large panel multistorey buildings, structural response of the prefabricated shear wall systems depend greatly on the shear deformability characteristics of the vertical joints between the wall panels.[1] In fact, the vertical joints form the shearing media connecting the adjoining wall panels. Considering the important role of vertical joints in the structural behaviour of prefabricated shear wall systems, an elaborate test programme was undertaken at the Central Building Research Institute to study the mechanism of failure of vertical joints and to derive the load carrying capacity of the joints under shear. The empirical formula for the load carrying capacity of the vertical joints was derived on the basis of shear-friction hypothesis and is given by the following equation:

$$P_f = 0.93A_k(\sqrt{f_c} + 1250\rho) \quad ..(1)$$

where, P_f, A_k and f_c are expressed in kg, cm² and kg/cm,² respectively. A detailed description of the test programme and discussion of the test results are reported elsewhere[2].

In the present paper, an attempt has been made to apply the principles of theory of plasticity to the fracture criterion of concrete for evaluation of the failure load of vertical shear key joints and compare them with the test results reported earlier.

Failure criterion of concrete

Many experimental studies on concrete have led to the conclusion that a modified Coulomb's yield criterion is suitable for fracture of concrete, *Fig* 1. In *Fig* 1(*a*), concrete is assumed to be incapable of carrying any tension and obeying Coulomb's yield criterion (straight line Mohr's envelope of failure) in compression. The requirement of zero tension is satisfied by terminating the circle as shown. The Mohr's envelope intersects the horizontal axis at an angle and the vertical axis at a distance *c* from the origin. In case, limited tensile strength of concrete is to be considered, Coulomb's modified yield criterion will apply as shown in *Fig* 1(*b*).

The yield criterion consists of two parts, namely, sliding and separation criteria. In the sliding criterion

$$|\tau| = c - \sigma\tan\phi \quad ....(2)$$

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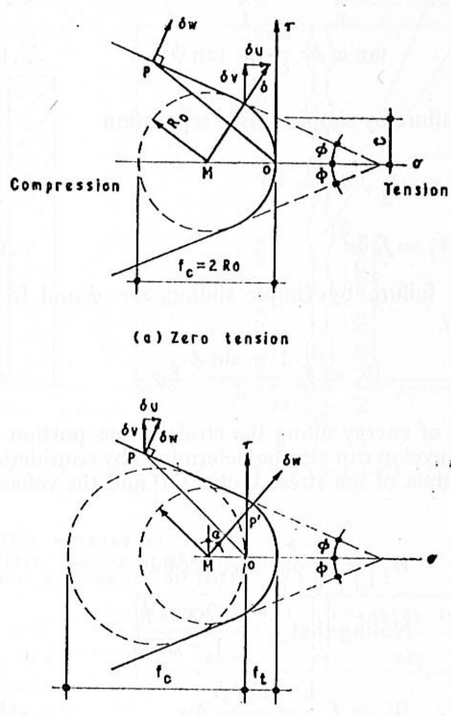
In case of separation criterion

$$\sigma_1 = f_t \quad ....(3)$$

For determining the yield criterion for concrete with limited tensile strength, three parameters, namely. the compressive strength, f_c, the tensile strength, f_t and the angle friction, ϕ should be known from experimental results.

Considering concrete as a rigid plastic material, it can be assumed that when concrete yields, the strains satisfy normality conditions. The expression for the rate of dissipation of energy can be derived in a straight forward manner on the assumption of perfect plasticity. Although it may not be rational, the approach is quite simplified. Superimposing the velocity coordinates on the stress coordinates as in *Fig* 1(*b*) the resultant of the tangential slip and normal separation components δv and δu across the failure surface is normal to the yield envelop. The rate of dissipation of energy per unit area W_i is given by



(a) Zero tension

(b) Small tension cut off

Fig 1 Modified Coulomb's criterion

NOTATIONS

A_k	= area of shear key	t	= thickness of shearing area
A_s	= area of reinforcement	w_e	= external work done
f_c	= characteristic strength of concrete	wi	= internal work done
f_t	= tensile strength of concrete	δ_u, δ_v	= velocity components in two perpendicular directions
f_y	= yield strength of reinforcement	δ_w	= resultant of the velocity components δ_u and δ_v
h	= height of the shearing area		
K_c, K_s	= constants	τ	= nominal shear stress
P_f	= failure load	σ	= direct stress
p	$= \dfrac{A_s}{A_k}$	ϕ	= angle of friction of concrete

the product of stress vector (σ, τ) and velocity vector (δu, δv). The stress vector is shown in *Fig* 1(*b*) as OP or OP′ and the resultant velocity vector as δw. Considering stress vector OP′, to be the sum of vectors OM and MP′, dissipation of energy per unit area is given as

$$W_i = R\delta w - (R - f_t)\delta u \qquad ..(4)$$

After substituting in equation (4)

$$R = \frac{f_c}{2} - f_t \frac{\sin\phi}{1 - \sin\phi} \qquad ..(5)$$

is obtained

$$W_i = \delta w\left(f_c \frac{1 - \sin\alpha}{2} + f_t \frac{\sin\alpha - \sin\phi}{1 - \sin\phi}\right) \qquad ..(6)$$

Also,

$$\tan\alpha = \frac{\delta u}{\delta v} \geqslant \tan\phi \qquad ..(7)$$

In case of failure by simple tensile separation

$$\alpha = \frac{\pi}{2}, \; \delta w = \delta u$$

therefore, $W_i = f_t\, \delta u$ ..(8)

In case of failure by simple sliding, $\alpha = \phi$ and from equation (6)

$$W_i = f_c \frac{1 - \sin\phi}{2} \delta w \qquad (9)$$

Dissipation of energy along the straight line portion of the failure envelop can also be determined by considering the dot product of the stress vector OP and the velocity δw

$$W_i = \overrightarrow{\mathrm{OP}\delta w} = c\cos\phi\,\delta w$$

$$\text{Noting that } f_c = \frac{2c\cos\phi}{1 - \sin\phi}$$

$$W_i = f_c \frac{1 - \sin\phi}{2} \delta w \qquad ..(10)$$

Chen and Drucker[3] have obtained equation (6) as described here. Jensen[4] has given an elaborate explanation of the modified Coulomb's yield criterion and has applied this in predicting the failure loads of plain construction joints, problems of shear and brackets.

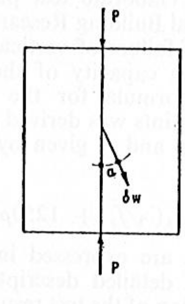


Fig 2 Shear failure of joint

Failure analysis of shear key joints

Vertical joints in large panel walls are shear key joints and subjected to shearing force. The shearing resistance across the shear keys and the horizontal reinforcing bars in the vertical joints provide the resistance to failure. Referring to *Fig* 2, the failure mechanism consists of a line of discontinuity between the loads *P* and right hand part of the shearing plane moves a distance δw in relation to the left hand one. Horizontal reinforcing bars which may be provided in the joint is assumed to resist forces only in its own direction. External work done by P

$$W_e = P\,\delta w \cos\alpha \qquad ..(11)$$

From equation (6), energy dissipated by concrete along the shear surface is

$$W_{ie} = \delta w\left(f_c \frac{1 - \sin\alpha}{2} + f_t \frac{\sin\alpha - \sin\phi}{1 - \sin\phi}\right) ht \qquad ..(12)$$

Internal work done by reinforcement

$$W_{ir} = A_s f_y\, \delta w \sin\alpha \qquad ..(13)$$

Assuming

$$\tau = \frac{P}{h_t} \qquad ..(14)$$

$$\text{and } \eta = \frac{A_s f_y}{h_t\, f_c} \qquad ..(15)$$

and equating external work to internal work

$$\frac{\tau}{c} = \frac{1 - \sin\alpha}{2\cos\alpha} + \frac{\sin\alpha - \sin\phi}{(1-\sin\phi)\cos\alpha}\frac{f_t}{f_c} + \eta\, tan\alpha \qquad ..(16)$$

This is upper bound solution. The minimum is found when

$$\frac{d\tau}{d\alpha} = 0$$

which gives

$$\sin\alpha = \frac{(1-\sin\phi) - 2\eta\,(1-\sin\phi) - \dfrac{2f_t}{f_c}}{(1-\sin\phi) - \dfrac{2f_t}{f_c}\sin\phi} \qquad ..(17)$$

The minimum value of τ may be obtained by substituting the value of α from equation (17) into equation (16).

In case of vertical joints tested for shear deformability it was found that stresses in the reinforcement at the initial stages were minimum and increased with further increase in loading. This may be due to the reason that as long as deformation in the joint was within tensile limits, stresses in the reinforcement were very small. The reinforcement in the joint starts to carry the load in its own direction, only when minute tensile cracks appeared in the joint. Hence, while applying the modified Coulomb's failure criterion, it is sufficient to take cognizance of tensile strength of concrete, only when there is no reinforcement across the failure surface. In a reinforced shear joint, it will not be too erroneous to neglect the tensile strength of concrete, $f_t = 0$. With this assumption equations (16) and (17) reduce to

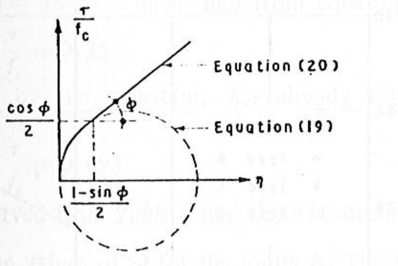


Fig 3 Coulomb's failure criterion in shear

$$\frac{\tau}{f_c} = \frac{1-\sin\alpha}{2\cos\alpha} + \eta\tan\alpha \qquad ..(18)$$

$$\text{and} \qquad \sin\alpha = 1 - 2\,\eta \qquad ..(19)$$

The minimum value of τ is obtained as

$$\frac{\tau}{f_c} = \sqrt{\eta\,(1-\eta)} \qquad ..(20)$$

Equation (20) is valid for $\alpha \geqslant \phi$

From equation (19)

$$\eta < \frac{1-\sin\phi}{2}$$

For the particular case when $\alpha = \phi$

$$\frac{\tau}{f_c} = \frac{1-\sin\phi}{2\cos\phi} + \eta\tan\phi \qquad ..(21)$$

Equation (21) is valid for $\eta \geqslant \dfrac{1-\sin\phi}{2}$

Equations (20) and (21) have been plotted as shown in *Fig* 3 in η and $\frac{\tau}{f_c}$ coordinate system. It may be noted that equation (20) is a circle with a radius equal to $\frac{1}{2}$ and its centre at $(\frac{1}{2}, 0)$ and equation (21) is a straight line tangential to the circle at the point with $(\eta,\ \tau/f_c)$ coordinates as $\left(\dfrac{1-\sin\phi}{2}, \cos\phi\right)$.

The details of the test panels tested at the Institute, are shown in *Fig* 4. A typical failure of the joint after test

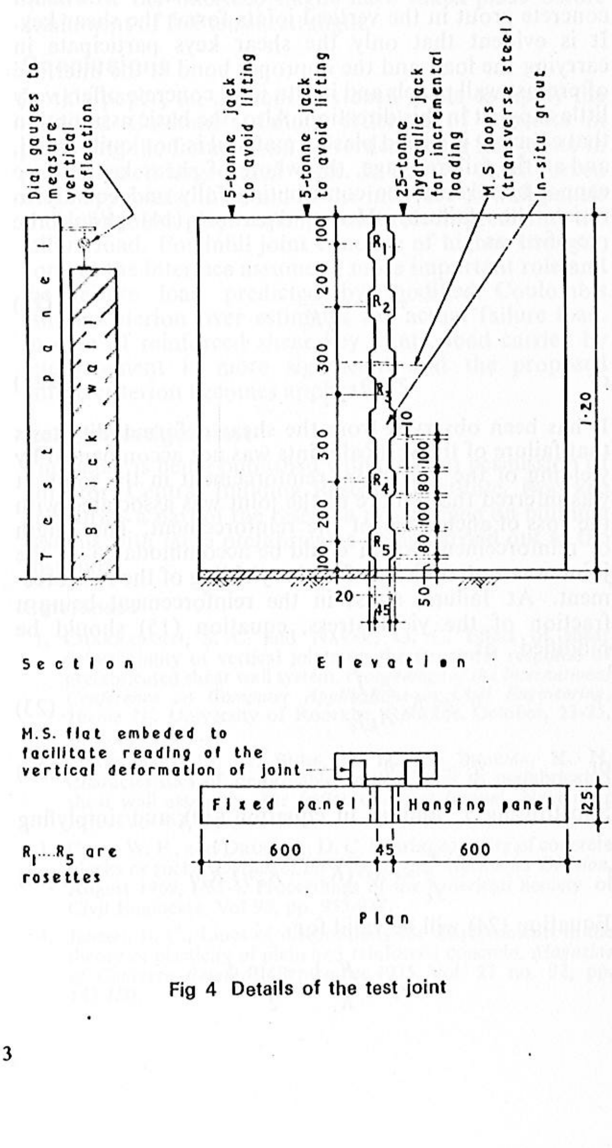


Fig 4 Details of the test joint

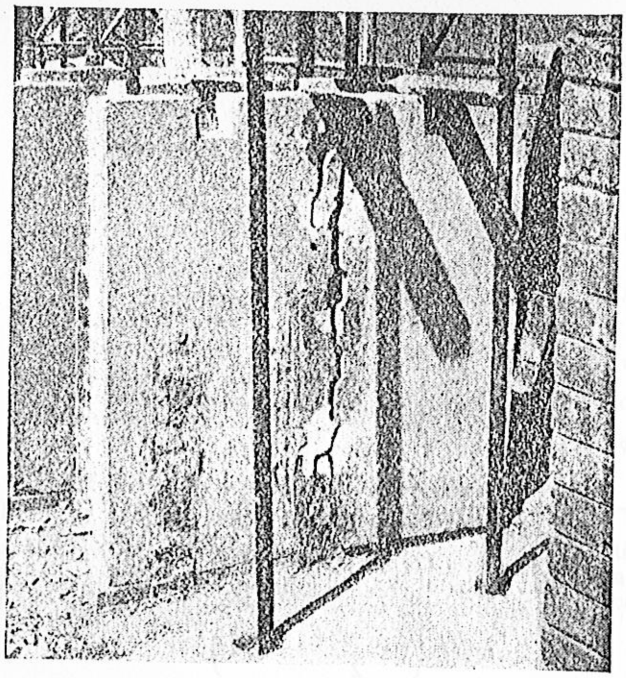


Fig 5 Typical failure of a joint

is shown in *Fig* 5. In all 23 joints were tested. The insitu concrete grout in the vertical joints forms the shear key. It is evident that only the shear keys participate in carrying the load and the improper bond at the interface of precast wall panels and in situ joint concrete offers very little support in this direction. Also, the basic assumption that concrete is a rigid plastic material is not quite valid, and at the failure stage, the whole of shear key surface cannot be taken to be contributing fully and equally to resist the failure. Hence equation (14) should be modified as

$$\tau' = \frac{P}{A_k K_c} \quad ..(22)$$

or

$$\tau' = \frac{\tau}{K_c} \quad ..(22a)$$

It has been observed from the shear deformability tests that failure of the vertical joints was not accompanied by yielding of the horizontal reinforcement in the joint. It was inferred that failure of the joint was associated with the loss of anchorage of the reinforcement. The length of reinforcement which could be accommodated in the joint was not sufficient for the yielding of the reinforcement. At failure, stress in the reinforcement being a fraction of the yield stress, equation (15) should be modified as

$$\eta' = K_s \frac{A_s f_y}{A_k f_c} \quad ..(23)$$

or

$$\eta' = K_s \eta \quad ..(23a)$$

Substituting τ' and η' in equation (20), and simplyfing

$$\frac{\tau}{f_c} = \sqrt{(K_c - K_s\eta)\, K_s\eta} \quad ..(24)$$

Equation (24) will be valid for

$$\eta \leqslant \frac{K_c}{K_s} \frac{1 - \sin\phi}{2}$$

Similarly, from equation (21)

$$\frac{\tau}{f_c} = K_c \frac{1 - \sin\phi}{2\cos\phi} + K_s \eta \tan\phi \quad ..(25)$$

The above equation is valid for

$$\eta \geqslant \frac{K_c}{K_s} \frac{1 - \sin\phi}{2}$$

Test results of the 23 joint specimens along with the relevant data are given in *Table* 1. It may be noted that Joints A are unreinforced whereas B, C and D are reinforced. Except joints D, which has 3 shear keys at equal spacing, joints A, B and C had 6 shear keys each.

The values of $\frac{\tau}{f_c}$ have been plotted for joints B, C and D against the respective values of η and shown in *Fig* 5. Through the test results of joints C and D for which the values of η are comparatively larger, a best fit straight line has been found and drawn in *Fig* 5. The equation for the straight line is given by

$$\frac{\tau}{f_c} = 0.0963 + 0.5344\,\eta \quad ..(26)$$

Assuming $\phi = 35°$ and comparing equations (25) and (26)

$K_c \approx 0.37$ and $K_s \approx 0.76$.

It can be seen that equation (24) is a circle in the $\left(K_s\eta, \frac{\tau}{f_c}\right)$ coordinate system having a radius equal to $\frac{K_c}{2}$ and with the centre at $\left(\frac{K_c}{2}, 0\right)$. Also equation (25) is tangent to equation (24) at

$$K_s\eta = K_c \frac{1 - \sin\phi}{2} \quad ..(26)$$

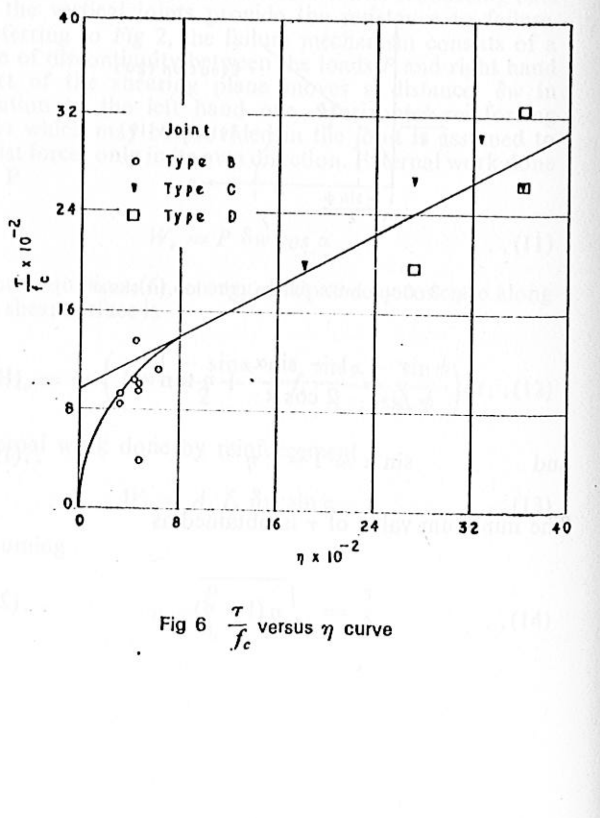


Fig 6 $\frac{\tau}{f_c}$ versus η curve

TABLE 1 Results of shear test of joints

Joint no	*Area of shear keys,* cm²	*Area of reinforcement,* cm²	*Failure load,* P_f kg	*Nominal shear stress* $\tau = \frac{P_f}{A_{k_2}}$ kg/cm²	f_c, kg/cm²	$\frac{\tau}{f_c}$	$\eta = \frac{A_s f_y}{A_k f_c}$
A–1	750	nil	6000	8.00	90	0.089	0
A–2	750	nil	7500	10.00	60	0.167	0
A–3	750	nil	5500	7.33	60	0.122	0
A–4	750	nil	5000	6.67	60	0.111	0
A–5	750	nil	9500	12.67	150	0.084	0
A–6	750	nil	9000	12.00	290	0.041	0
A–7	750	nil	16000	21.33	290	0.073	0
A–8	750	nil	5500	7.33	180	0.041	0
B–1	750	1.68	10000	13.33	130	0.103	0.045
B–2	750	1.68	11500	15.33	180	0.085	0.032
B–3	750	1.68	13250	17.67	130	0.135	0.045
B–4	750	1.68	7500	10.00	90	0.111	0.065
B–5	750	1.68	8500	11.33	120	0.094	0.049
B–6	750	1.68	3500	4.67	120	0.039	0.049
B–7	750	1.68	9000	12.00	120	0.100	0.049
B–8	750	1.68	12500	16.67	180	0.093	0.032
C–1	750	4.71	8900	11.87	45	0.264	0.363
C–2	750	4.71	12000	16.00	60	0.267	0.272
C–3	750	4.71	11400	15.20	50	0.304	0.327
C–4	750	4.71	13300	17.73	90	0.197	0.181
D–1	375	4.71	8900	23.73	90	0.264	0.363
D–2	375	4.71	11000	29.33	90	0.326	0.363
D–3	375	4.71	8800	23.47	120	0.196	0.363

Equation (24) has been plotted with the values of K_c and K_s as obtained from equation (26). The curve obtained gives a fair representation of the failure envelop of the test results of joints B. Evaluation of the experimental constants K_c and K_s are quite rational and realistic since these are based on the results of test joints with values of η for which straight line failure envelop is applicable [for values of $\eta \geqslant 0.102$ as obtained from equation (29).]

In case of unreinforced joints A, $\eta = 0$. Assuming, $\frac{f_t}{f_c} = 0.1$, for the minimum value of $\frac{\tau}{f_c}$ from equation (17)

$$\sin \alpha = \frac{1 - \sin \phi - 2\frac{f_t}{f_c}}{1 - \sin \phi - 2\frac{f_t}{f_c} \sin \phi} \quad ..(28)$$

Assuming $\phi = 35°$, $\alpha = 46.6°$ and from equation (16)

$$\frac{\tau}{f_c} = 0.25$$

Multiplying by the constant, K_c, already obtained earlier

$$\frac{\tau}{f_c} = 0.093$$

It may observed from *Table* 1 that there is considerable scatter in the values of $\frac{\tau}{f_c}$ for the joints A. The scatter may be understood if it is realised that the bond between the old concrete in the wall panel and the new concrete in the joint depends on a number of technological factors like homogenity of concrete in the joint, surface texture of the wall panels, control of water-cement ratio in the infill concrete, prevention of shrinkage, etc. Due to imperfect bond, it may be possible that bond at the interface fails before failure of infill concrete in tension. Moreover, the assumption of constant value of the angle ϕ and $\frac{f_t}{f_c}$ ratio for different compressive strength of infill concrete is not considered to be very rational. It may be noticed that for joints A-5 to A-8 where the strength of infill concrete is comparatively higher, $\frac{\tau}{f_c}$ ratios are lower. This may be due to the reason that bond failure at the interface might have taken place before attainment of full tensile strength.

Conclusions

In this paper, an attempt has been made to apply the modified Coulomb's failure criterion of concrete in predicting the failure load of vertical shear key joints in prefabricated large panel walls. In case of unreinforced shear key joints, the bond at the interface of wall panels and infill joints plays an important role in predicting the failure load. For infill joint concrete of higher strength, bond at the interface assumes a more important role and the failure load predicted by modified Coulomb's failure criterion over estimates the actual failure load. In case of reinforced shear key joints, load carried by reinforcement is more significant and the proposed failure criterion becomes applicable.

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