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Scattering of Sunlight in the Terrestrial Atmosphere

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ABSTRACT : The sun's radiant energy, while passing through the earth's atmosphere, is depleted in intensity due to scattering and absorption by its constituents. Two types of scattering take place by these elements. When the particle radii are much smaller compared to the wavelength, as in the case of air-molecules ($r \leqslant 0.1\lambda$), Rayleigh scattering applies. In the case of large solid and liquid particles suspended in the atmosphere called aerosols ($r \simeq 0.04\mu$ to 10μ), more complicated theory developed by Mie is applicable. But in practical calculations an analytical formula provided by Angstrom is frequently used because it seems to be simple and accurate enough for the purpose. The aim of the present paper is to discuss the two types of scattering of solar electromagnetic radiation and to provide computed values of spectral transmission factors for various parameters. These are expected to be useful for application oriented problems.

1. INTRODUCTION

Computational methods are usually adopted in estimating direct beam solar irradiance for clear sky conditions on the earth's surface particularly in applied problems. The energy emitted by the sun is substantially reduced in intensity before it reaches the earth's surface. This reduction is caused due to interaction of spectral energy with the constituents of the terrestrial atmosphere. Therefore to calculate the intensity of direct beam solar irradiance on the earth's surface, the atmospheric transmission factors due to scattering and absorption by different atmospheric elements should be known with certain degree of accuracy. The aim of this paper is to discuss the scattering of sun's radiant energy in the atmosphere. Two types of scattering take place by the atmospheric elements, namely, molecular and large particle. When the particle radii are much smaller compared to wavelength, i.e., $r \leqslant 0.1\lambda$, Rayleigh scattering applies. Air molecules fulfill this condition. In the case of large solid and liquid particles suspended in the lower atmosphere (aerosols), the particle radii are much larger (0.04μ to 10.0μ). In this case more complicated theory developed by Mie [1] is applicable. But due to complexity of Mie's theory, an analytical formula given by Angstrom [2] is generally used in practical calculations because it is simple and accurate enough. In this paper, the results of calculations of spectral transmission factors for both the molecular and aerosol scattering for wavelengths from 0.2μ to 9.0μ are given. Solar electromagnetic radiation is mostly confined in this range [3]. The total spectral transmission factors for combined influence of molecular and aerosol scattering for different solar altitudes, station heights above MSL and Angstrom's turbidity coefficient are tabulated.

2. ATTENUATION OF THE DIRECT SOLAR IRRADIANCE IN THE BATH'S ATMOSPHERE

The Direct solar beam in passing through the earth's atmosphere is depleted in intensity due to scattering and absorption by constituents of the atmosphere such as airmolecules, aerosols, ozone, water vapour and carbon di-oxide etc. The total depletion of solar energy at a given wavelength λ is characterised by over-all extinction coefficient a_λ which takes into account both scattering and absorption coefficient.

Let us consider a parallel beam of monochromatic radiation incident on a single particle. Scattering is said to occur if all the energy attenuated from the original beam is present in the radiation field surrounding the particle. Absorption is said to occur if the sum of the transmitted and the scattered energies integrated over all the scattering angles is less than the incident energy.

According to Beer's law the loss dI_λ in the intensity I_λ of a parallel monochromatic beam is given by the formula :

$$dI_\lambda = I_\lambda a'_\lambda dx \qquad (1)$$

where dx is the pathlength and a'_λ is the extinction coefficient which is the sum of all the scattering and absorption coefficients. The integration of Eqn. (1) yields :

$$I_\lambda = I_{o\lambda} \, exp \, (-\int_0^\infty a'_\lambda \, dx) \qquad (2)$$

3. Rayleigh Scattering

As stated earlier Rayleigh scattering applies in the case of air molecules whose radii are much smaller in comparison with the wavelength, i.e., $r \leqslant 0.1\lambda$. The spectral radiation intensity after depletion by airmolecules of the entire terrestrial atmosphere can be written as :

$$I_{a\lambda} = I_{o\lambda} \, exp \, (-\sigma_{a\lambda} \cdot H . m_h(0)) \qquad (3)$$

Where $I_{a\lambda}$ and $I_{o\lambda}$ are the spectral energy at the wavelength λ at the earth's surface and outside the earth's atmosphere, respectively, $\sigma_{a\lambda}$ is the volume scattering coefficient per metre for airmolecules, $H(=7991$ metres) is the vertical height of imaginary homogeneous atmosphere, which is equivalent to actual atmosphere at normal temperature and pressure (NTP), $m_h(0)$ is the absolute actual air mass which takes into account the path length of sun rays at solar altitude 0 relative to zenith. At high altitude stations the number of airmolecules encountered by sun rays decreases as also the value of $m_h(0)$, which is given by

$$m_h(0) = \frac{p_h}{p_o} m_r(0) \qquad (4)$$

Where p_h and p_o are the atmospheric pressures at the station height h and o (mean sea level) respectively. $m_r(0)$ is the relative airmass at the solar altitude θ. At lower altitudes of the sun the cosecant gives values of $m_r(0)$ which are increasingly too high because of errors due to atmospheric refraction, curvature of the earth etc. Therefore relative airmasses based on the widely used computations of Bemporad [4] have been taken for the present calculations.

Now the Eqn. (3) can be written as

$$I_{a\lambda} = I_{o\lambda} \tau_{a\lambda}^{m_h(\theta)} \qquad (5)$$

Where $\tau_{a\lambda}$ is the zenith spectral transmissivity or transmission factor and the transmissivity for the solar altitude of θ is calculable by raising it with the power of $m_h(\theta)$.

The Rayleigh volume scattering coefficient $\sigma_{a\lambda}$ at particular wavelength λ is given by

$$\sigma_{a\lambda} = \frac{32\pi^2(n_\lambda - 1)}{3\lambda^4 N} \left[\frac{6 + 3\rho_{n_\lambda}}{6 - 7\rho_{n_\lambda}}\right] \qquad (6)$$

Here N is the number density (number of molecules /cm^3), n_λ the relative index of refraction for the medium and ρ_{n_λ} is the depolarization factor. The latest values on volume scattering coefficient are given by Penndorf [5]. There is good reason to believe that these values are more accurate. These have been used for the calculations presented in this paper.

4. Large Particle or Mie Scattering

In the case of large solid and liquid particles suspended in the atmosphere, known as aerosols, the Rayleigh scattering theory is not applicable. The radii of the bulk of the aerosols are found to lie in the range from about 0.04μ to 10.0μ. In the case of such particles a theory developed by Mie is applicable. According to Mie's theory the scattering coefficient $\sigma_m(r,\lambda)$ of the individual particles can be expressed in terms of dimensionless scattering cross-section $K(\alpha', n_\lambda)$ which is a function of the size parameter $\alpha' = \frac{2\pi r}{\lambda}$ and refractive index n_λ. Thus for the aerosol particles the volume scattering coefficient $\sigma_{D\lambda}$ can be written as

$$\sigma_{D\lambda} = \int_0^\infty \sigma_m(r,\lambda) \, dN(r) = \int_0^\infty \pi r^2 K(\alpha', n_\lambda) dN(r) \qquad (7)$$

Where $N(r)$ is the number of particles per unit volume within the interval dr of the radius r. The expression for scattering cross-section is

$$K(\alpha', n_\lambda) = \frac{2}{\alpha'^2} \sum_1^\infty (2m+1) \left[|a_m|^2 + |b_m|^2\right] \qquad (8)$$

Here a_m and b_m are the Mie amplitude functions of electric and magnetic partial waves. For calculating the total extinction caused due to aerosols their density and size distribution and vertical extent of their presence in the atmosphere must be precisely known alongwith the scattering cross-sections over the entire range of size parameters, Nevertheless these variables are inadequately known. In view of this Angstrom's analytical formula which is simple and accurate enough for practical computation purposes, is generally used.

5. ANGSTROM'S FORMULA

According to this analytical formula the scattering coefficient, $\sigma_{\beta\lambda}$, for aerosols is given by

$$\sigma_{\beta\lambda}=\beta\lambda^{-\alpha} \quad (9)$$

Here β is called Angstrom's turbidity coefficient and α the wavelength exponent which is closely related to the size of the scattering particles and to the frequency of their size parameters. In the present calculations α has been taken constant and equal to 1.3. This empirical result has been theoretically verified by investigations of several workers [6-9.].

The spectral solar intensity, $I_{\beta\lambda}$, at the earth's surface due to aerosol scattering can be written as

$$I_{\beta\lambda}=I_{o\lambda}\ exp\ (-\sigma_{\beta\lambda}\cdot m_r(\theta))=I_{o\lambda}\tau_{\beta\lambda}^{m_r(\theta)} \quad (10)$$

Where $\tau_{\beta\lambda}$ is the zenith spectral transmissivity of the atmosphere due to aerosol scattering and $\tau_{\beta\lambda}^{m_r(\theta)}$ gives transmissivity at solar altitude θ. It may be mentioned here that β is available for a number stations throughout the world [10,11]

6. COMBINED INFLUENCE OF MOLECULAR AND AEROSOL SCATTERING

The spectral transmission factors due to the combined influence of the two types of scattering can be obtained, according to Bouger-Lambert law, by means of the expression

$$\tau_{s\lambda}(\theta)=\tau_{a\lambda}^{m_h(\theta)}\tau_{\beta\lambda}^{m_r(\theta)} \quad (11)$$

The values of $\tau_{a\lambda}^{m_h(\theta)}$, and $\tau_{\beta\lambda}^{m_r(\theta)}$ and $\tau_{s\lambda}(\theta)$ have been calculated for 63 wavelengths in the range 0.20μ to 9.0μ. The computations were carried out for solar altitudes of 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 75, 80 and 90 degrees. The values of H considered were 0, 3000 and 5000 m. Angstrom's turbidity was varied from 0.05 to 0.30 at an interval of 0.05. Due to

TABLE 1—RAYLEIGH SPECTRAL TRANSMISSION FACTORS FOR DIFFERENT STATION HEIGHTS AND SOLAR ALTITUDES

Wavelength	H=0 m				H=5000 m			
	Solar Altitude							
(micron)	90	45	30	15	90	45	30	15
0.20	0.0005	0.0000	0.0000	0.0000	0.0096	0.0014	0.0001	0.0000
0.26	0.1031	0.0406	0.0106	0.0002	0.2217	0.1202	0.0500	0.0035
0.30	0.2438	0.1792	0.0873	0.0095	0.4235	0.2978	0.1794	0.0376
0.35	0 5305	0.4091	0.2814	0.0888	0.6466	0.5407	0.4180	0.1890
0.40	0.6956	0.5994	0.4838	0.2499	0,7823	0.7074	0.6120	0.3915
0.45	0.8001	0.7303	0.6402	0.4266	0.8616	0.8.06	0.7424	0.5661
0.50	0.8653	0.8154	0.7487	0.5758	0.9087	0.8738	0.8258	0.6938
0.55	0.9066	0.8709	0.8220	0.6876	0.9377	0.9133	0.8793	0.7822
0.60	0.9335	0.9076	0.8714	0.7689	0.9562	0.9387	0.9142	0.8426
0.70	0.9638	0.9494	0.9290	0.8688	0.9751	0.9651	0.9508	0.9082
0.80	0.9788	0.9702	0.9580	0.9213	0.9832	0 9764	0.9667	0.9373
0.90	0.9867	0.9813	0.9736	0.9503	0.9870	0.9817	0.9742	0.9513
1.0	0.9913	0.9878	0.9827	0.9672	0.9919	0.9886	0.9839	0.9694
1.4	0.9977	0.9968	0.9955	0.9914	0.9974	0.9964	0.9949	0.9903
1.8	0.9992	0.9988	0.9983	0.9968	0.9992	0.9989	0.9984	0.9969
2.2	0.9998	0.9997	0.9996	0.9991	0.9997	0.9995	0.9993	0.9987
2.8	0.9998	0.9997	0.9996	0.9992	0,9999	0.9998	0.9997	0.9995
3.0	0.9999	0.9998	0.9998	0.9996	0.9999	0.9998	0.9997	0.9995

TABLE 2—AEROSOL SPECTRAL TRANSMISSION FACTORS FOR DIFFERENT ANGSTROM'S TURBIDITY COEFFICIENTS AND SOLAR ALTITUDES

Wavelength (micron)	$\beta=0.05$ Solar Altitude 90	45	30	15	$\beta=0.10$ Solar Altitude 90	45	30	15	$\beta=0.20$ Solar Altitude 90	45	30	15
0.20	0.6669	0.5648	0.4447	0.2127	0.4447	0.3190	0.1978	0 0453	0.1978	0.1018	0.0391	0.0020
0.26	0.7479	0.6662	0.5621	0.3327	0.5620	0.4438	0.3159	0.1107	0.3159	0.1970	0.0998	0.0123
0.30	0.7873	0.7137	0.6198	0.4011	0.6198	0.5094	0.3842	0.1609	0.3842	0.2595	0.1476	0.0259
0.35	0.8222	0.7588	0.6761	0.4734	0.6760	0.5758	0.4570	0.2241	0.4570	0.3315	0.2089	0.0502
0.40	0.8483	0.7929	0.7196	0.5334	0.7196	0.6287	0.5178	0.2845	0.5178	0.3453	0.2681	0.0809
0.45	0.8683	0.8195	0.7540	0.5831	0.7540	0.6716	0.5685	0.3401	0.5685.	0.4510	0.3232	0.1156
0.50	0.8842	0.8406	0.7817	0.6248	0.7817	0.7067	0.6111	0.3904	0.6111	0.4994	0.3735	0.1520
0.55	0.8969	0.8578	0.8045	0.6600	0.8045	0.7359	0.6472	0.4356	0.6472	0.5415	0.4189	0.1897
0.60	0.9074	0.8720	0.8234	0.6900	0.8234	0.7604	0.6780	0.4761	0.6780	0.5782	0.4597	0.2267
0.70	0.9235	0.8940	0.8530	, 0.7381	0.8530	0.7992	0.7276	0.5448	0.7276	0.6387	0.5294	0.2968
0.80	0.9354	0.9101	0.8749	0.7747	8.8749	0.8282	0.7654	0.6002	0.7654	0.6859	0.5859	0.3602
0.90	0,9443	0.9223	0.8916	0.8033	0.8916	0.8507	0.7950	0.6453	0.7950	0.7237	0.6321	0.4163
1.0	0.9512	0.9319	0.9049	0.8261	0.9048	0.8685	0.8187	0.6825	0.8187	0.7543	0.6703	0.4658
1.4	0.9682	0.9553	0.9375	0.8840	0.9375	0.9130	0.8788	0.7814	0.8788	0.8335	0.7724	0.6106
1.8	0.9770	0.9677	0.9545	0.9149	0.9545	0.9364	0.9110	0 8370	0.9110	0.8769	0.8300	0.7006
2.2	0.9822	0.9750	0.9648	0.9334	0.9647	0.9507	0.9308	0.8719	0.9307	0.9038	0.8663	0.7602
2.6	0.9857	0.9798	0.9715	0.9463	0.9715	0.9601	0.9438	0.8956	0.9489	0.9287	0.9004	0.8184
3.0	0.9880	0.9832	0.9763	0.9552	0.9763	0.9668	0.9530	0.9125	0.9532	0.9346	0.9086	0.8326

TABLE 3—TRANSMISSION FACTORS DUE TO COMBINED INFLUENCE OF MOLECULAR AND AEROSOL SCATTERING

Wavelength (micron)	H=O m, $\beta=0.10$ Solar Altitude 90	45	30	15	H=5000 m, $\beta=0.05$ Solar Altitude 90	45	30	15
0.20	0.0002	0.0000	0.0000	0.0000	0.0064	0.0008	0.0000	0.0000
0.26	0.0569	0.0180	0.0033	0.0000	0.1658	0.0801	0.0281	0.0012
0.30	0.1511	0.0913	0.0335	0.0015	0.3334	0.2125	0.1112	0.0151
0.35	0.3586	0.2356	0.1286	0.0199	0.5316	0.4103	0.2826	0.0895
0.40	0.5006	0.3768	0.2505	0.0711	0·6636	0.5609	0.4404	0.2088
0.45	0.6033	0.4905	0.3640	0.1451	0.7481	0.4184	0.5598	0.3301
0.50	0.6764	0.5762	0.4575	0.2248	0.8035	0.7345	0.6455	0.4335
0.55	0.7294	0.6409	0.5320	0.2995	0.8410	0.7834	0.7074	0.5163
0.60	0.7686	0.6901	0.5908	0 3661	0.8677	0.8185	0.7528	0.5814
0.70	0.8221	0.7588	0.6759	0.4733	0.9005	0.8628	0.8110	0.6703
0.80	0.8564	0.8035	0.7333	0.5530	0.9197	0.8886	0.8458	0.7261
0.90	0.8797	0.8348	0.7740	0.6132	0.9320	0.9054	0.8686	0.7642
1.0	0.8969	0.8579	0.8045	0.6601	0.9435	0.9213	0 8903	0.8008
1.4	0.9353	0.9100	0.8748	0.7747	0.9657	0.9521	0.9327	0.8754
1.8	0.9537	0.9353	0.9095	0.8343	0.9762	0.9666	0.9530	0.9120
2.2	0.9645	0.9504	0.9304	0.8711	0.9819	0.9745	0.9641	0.9322
2.6	0.9713	0.9598	0.9434	0.8949	0.9856	0.9796	0.9712	0.9458
3.0	0.9762	0.9666	0.9528	0.9120	0.9879	0.9830	0.9760	0.9547

scarcity of space it is difficult to present all the computed data. However, Table 1, 2 and 3 give some abridged results of these calculations. An idea of the nature of variation of these transmission factors can be had from Fig. 1. It can be seen that spectral transmissivities due to Rayleigh scattering become almost equal to unity at wavelengths about 1.6μ at all solar altitudes. The aerosol transmissivities are much lower and their variation becomes slow only above about 3.0μ.

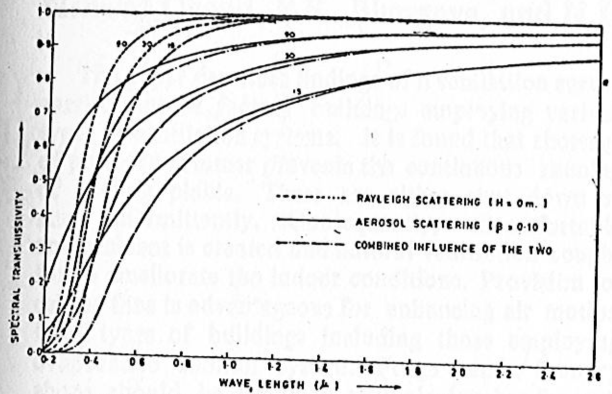

FIG. 1 TYPICAL SPECTRAL TRANSMISSIVITIES AT SOLAR ALTITUDES OF 15, 30 AND 90 DEGREES.

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