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Behaviour of reinforced lightweight concrete beams in flexure and shear

B. K. Jindal

Lightweight concrete beams designed according to the elastic theory were tested under third point loading. The ultimate loads and deflections were calculated and compared with the experimental values. Another set of beams was tested for shear strength. Beams of dense concrete of equivalent strength were also tested for the sake of comparison. The test results of both the lightweight and the dense concrete beams under flexure showed that their load factors were similar. However, the flexural rigidity of the lightweight concrete beams was comparatively less — the span-deflection ratios were greater than the minimum acceptable value of 300, and their shear strength was found to be 75 to 90 per cent of the dense concrete beams.

In continuation of investigations on the development and use of lightweight aggregates, a study of the behaviour of reinforced lightweight concrete members was undertaken. In the first instance, the performance of beams under flexure and shear was investigated. In this paper, the behaviour of dense and lightweight concrete beams is compared.

Materials

The lightweight aggregate used in the investigation was a sintered fly ash aggregate produced at the Central Building Research Institute on a pilot plant[1]. This material, graded from ¾ to ⅜ in, was used as the coarse fraction of the aggregate, while either crushed sintered aggregate or natural sand was used as fine aggregate, the grading conforming to IS : 383-1962. Gravel was used for dense concrete. The physical properties of the aggregates used are given in *Table* 1. The cement used was ordinary Portland cement conforming to IS : 269-1958. Round mild steel bars conforming to IS : 432-1953 were used for reinforcement.

The proportioning of the concrete mixes was carried out on weight basis. The concrete was mixed in a 10/7 ft³ mixer. In view of the high water absorption of lightweight aggregate, it was first wetted in the mixer by adding to it a part of the mixing water, the mixer being given a few revolutions to distribute the water uniformly. The cement and the remaining water were then added and the entire mass mixed thoroughly for about five minutes. In mixing dense concrete, all the ingredients were mixed dry and a predetermined amount of water was then added. The workability was controlled in either case.

The properties of structural lightweight concretes using sintered fly ash aggregate and dense gravel concretes of comparable strengths have been reported earlier[2]. To study and compare the behaviour of reinforced lightweight concrete beams, the following concretes were selected :

- 1 : 1.3 : 2.7 all-lightweight concrete aggregate
- 1 : 1.3 : 2.7 lightweight concrete containing natural sand
- 1 : 1.7 : 3.3 dense gravel concrete.

TABLE 1 The physical properties of aggregates used in the investigation

Tests	Coarse aggregate		Fine aggregate	
	Gravel	*Sintered fly ash*	*Sand*	*Crushed sintered fly ash*
Sieve analysis, per cent passing				
¾ in	94·7	100		
⅜ in	15·7	50		
$\frac{3}{16}$ in	2·2	0		
B. S. No. 7			97·6	71·5
14			89·4	51·5
25			40·2	39·3
52			4·9	30·1
100			0·8	20·0
Fineness modulus	6·38	6·50	2·67	2·86
Bulk density, lb/ft³	105	42	90	63
Water absorption by volume	0·74	17·7	—	—
Bulk specific gravity gm/cm³	2·68	1·37	—	—

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TABLE 2 Physical properties of sintered fly ash and gravel concretes

Mix by volume	*Water-cement ratio by weight*	*Compaction factor*	*Cement content per 100 ft,³ bags*	*Dry density lb/ft³*	*28-days strengths, lb/in²*					*Modulus of elasticity, lb/in² × 10⁶*
					Cube	*Cylinder*	*Tensile*	*Flexural*	*Bond*	
				All-lightweight aggregate concrete						
1 : 1·3 : 2·7	0·70	0·72	24·0	93·5	3098	2302	332	491·5	496	1·89
				Lightweight concrete containing sand						
1 : 1·3 : 2·7	0·61	0·746	25·0	105·3	3318	2422	334	481	490	2·34
				Gravel concrete						
1 : 1·7 : 3·3	0·51	0·83	18·9	145·0	3289	2415	356	508	510	4·21

These mixes were selected to give a minimum 28-days compressive strength of 3,000 lb/in² in the field under good supervision and gave actual strengths of 4,298, 4,690, and 4,648 lb/in², respectively under laboratory conditions. Their physical properties under field conditions of curing are given in *Table* 2.

Design of beams for flexural tests

The British and American Codes of Practice recommend the use of both the conventional method of design based on modular ratio and the ultimate strength or load factor method[3, 4]. In the present investigation the conventional method was selected because of its widespread usage. The reinforced concrete beams were designed for a superimposed load of 7 tons distributed equally at the third points of a 9-ft span. The British and Indian codes of practice require that a factor of safety of 3 be applied to the cube strength for concrete in compression [3, 5]. Accordingly, the safe permissible compressive stress c was taken as 1,000 lb/in² for all the mixes. The safe permissible tensile stress in the steel reinforcement t was taken as 18,000 lb/in² [5]. According to the elastic theory, the depth coefficient of the neutral axis N of a concrete flexural member depends upon the modular ratio m and permissible stresses in concrete and steel. The modular ratio for lightweight concrete is more than that for dense concrete. The value of N and the resistance moment factor Q of the former will, therefore, be more than that of the latter[2]. In the present investigation the instantaneous value of the modulus of elasticity of concrete was taken for computing the design constants. However, for the same superimposed loads, lightweight concrete flexural members will be thinner in section whether the instantaneous or the effective modulus of elasticity of the concrete is taken.

The detailed design for the all-lightweight aggregate concrete flexural beam designated *A* is given in *Appendix* 1. The beams made with lightweight concrete containing sand and with gravel concrete, designated *B* and *C*, respectively, were also designed in a similar manner. The salient design features of all the beams are reported in *Table* 3.

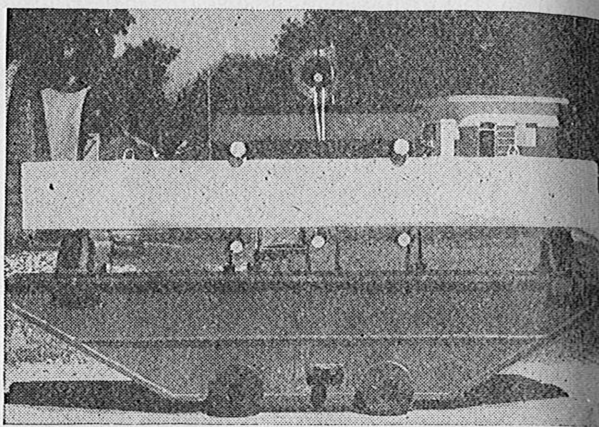

Fig 1 Test arrangement for the flexural tests of the beams

Fabrication

The beams were cast in a rigid wooden frame. The reinforcement, extending to within 1·5 in from the ends of the beam, was tied into a rigid cage before it was placed in the forms. Two batches of concrete were required for each beam and at least three 4-in cubes were cast from each batch. An internal vibrator was used to compact the concrete. Demoulding was done one day after casting and the specimens were subsequently cured under wet gunny bags for 28 days.

Testing

The beams were tested in a 500 tons capacity Lesenhausenwerk building materials testing machine, but the load was recorded on a calibrated 50 tons proving ring. The beams were simply supported over a span of 9 ft and tested under third point loading. This has the advantage of combining two different test conditions *viz*, bending without shear force being present between the two loads, and constant shear force in the two end sections (*Appendix* 1). Dial gauges were fixed—two each at the midspan and at the loading points—to record the deflections at different loads (*Fig* 1). The beams were whitewashed so that the crack pattern could be easily seen and marked on the beam during the test.

The load was applied in increments of 1 ton upto 8 tons, and thereafter in increment of 2 tons upto

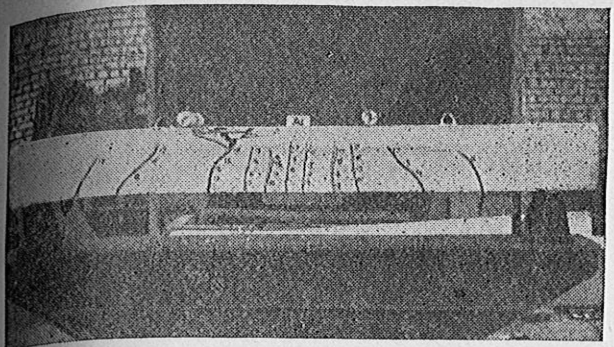

Fig 2 Showing the flexural failure of a lightweight concrete beam

destruction. After each increment, the deflection readings were taken and the surface cracks carefully marked.

Vertical cracks are expected to form in the central 3-ft portion of the beam as this region is subjected to constant bending moment and has no shear force. Under gradually increasing load minor cracks appeared before working load was reached. At working loads these cracks were very fine and comparable with those in dense concrete beams; as such they were not of much significance. As loading was continued, the cracks increased in width and length, thereby decreasing the area of the compression zone. The compressive stress in the concrete and the tensile stress in the steel continued to increase not only due to the increase in the load, but also due to the reduction in the area of the compression zone, until a rapid increase in the deflection of the beam indicated that the steel had reached the yield point. The stress in the concrete then increased rapidly till it reached the compressive strength of the concrete and the destruction of the compression zone brought the flexural failure of the beam (*Fig* 2). Such a failure is called a tension failure.

Ultimate loads

The ultimate load carrying capacity of a reinforced concrete beam can be predicted on the basis of the ultimate strength theory. It has been shown that when a beam subjected to flexure reaches failure, the concrete stress in the compression zone has a curved distribution across the cross-section[6, 7]. The stress conditions at ultimate load of a rectangular concrete beam (with tension reinforcement only) subjected to flexure is shown in *Fig* 3. The notation is as follows:

b = width of rectangular beam

d = distance from centroid of tension reinforcement to compression edge of beam

A_s = area of tension reinforcement

C = total internal compressive force in concrete

T = total tensile force in steel reinforcement

a = distance from neutral axis to compression edge of beam

e_u = ultimate strain in concrete

e_y = steel strain at yielding

f_c = compressive strength of 6 in × 12 in concrete cylinders

f_{su} = stress in tensile reinforcement at ultimate load

f_y = steel stress at yielding

k_1, k_2 and k_3 = coefficients related to the magnitude and position of internal compressive force in concrete compression zone

k_u = a/d ratio indicating position of neutral axis

M_u = ultimate bending moment

$p_b = \dfrac{A_s}{bd}$ for balanced section

k_1, k_2 and k_3 are given by the following equations:

$$k_1 = 0 \cdot 94 - \frac{f_c}{26{,}000} \quad \ldots\ldots (1)$$

$$k_2 = 0 \cdot 50 - \frac{f_c}{80{,}000} \quad \ldots\ldots (2)$$

$$k_3 = \frac{3{,}900 + 0 \cdot 35 f_c}{3{,}000 + 0 \cdot 82 f_c - \dfrac{f_c^2}{26{,}000}} \quad \ldots\ldots (3)$$

TABLE 3 Design features of beams for flexural test

c = permissible compressive stress in concrete, lb/in²
t = permissible tensile stress in steel, lb/in²
m = modular ratio
N = depth coefficient of neutral axis
J = coefficient of lever arm
Q = Resistance moment factor
b = width of beam
d = effective depth of beam
A_s = tensile reinforcement, in²
p = steel reinforcement, per cent
E = modulus of elasticity, lb/in²
I = moment of inertia on cracked section basis, in⁴

Designation of beam	*c*	*t*	*m*	*N*	*J*	*Q*	*bd*	A_s	*p*	*E*	*I*	*EI, lb/in²×10⁶*
					All-lightweight aggregate concrete							
A	1000	18000	15·9	0·47	0·843	198	9×13·0	1·516	1·29	1·89	1826	3451
					Lightweight concrete containing sand							
B	1000	18000	12·8	0·415	0·862	179	9×13·5	1·516	1·25	2·34	1754	4104
					Gravel concrete							
C	1000	18000	7·15	0·284	0·905	128	10×15·5	1·224	0·79	4·21	1360	5725

The equilibrium of force and moments acting on a section of the beam is expressed by the following equations :

$$C = k_1\ k_3\ f_c\ b\ a = T = A_s\ f_{su} \quad \ldots\ldots(4)$$

$$M_u = T\ (d - k_2 a) = A_s\ f_{su}\ (1 - k_2 k_u)d \quad \ldots\ldots(5)$$

or $M_u = C\ (d_2 - \ k_2 a)$

$$= k_1 k_3\ f_c\ ba\ (1 - k_2 k_u)\ d$$

$$= k_1 k_3\ k_u\ f_c\ (1 - k_2 k_u) bd^2 \quad \ldots\ldots(6)$$

For a failure on account of simultaneous yielding of steel and crushing of concrete,

$$C = k_1 k_3 f_c\, k_u\, bd = T = A_s f_y \quad \ldots\ldots(7)$$

$$\text{and } k_u = \frac{e_u}{e_y + e_u} \quad \ldots\ldots(8)$$

Taking e_u and e_y equal to 0·3 and 0·15 per cent, respectively, $k_u = \frac{0\cdot3}{0\cdot3 + 0\cdot15} = 0\cdot667$

Knowing the value of k_u, C and T, M_u can be obtained either from equation 5 or 6.

From equation 7,

$$p_b = k_1 k_3\ k_u \frac{f_c}{f_y}$$

$$= 0\cdot667\ \ k_1 k_3 \frac{f_c}{f_y}$$

Taking the value of f_c = 2,250 lb/in²

and f_y = 40,000 lb/in²

we get p_b = 0·032

In our case the maximum value of p is 0·0129 (*Table* 3). Therefore, all the beams in the present investigations were under-reinforced from the point of view of the ultimate strength theory and their failure was initiated by the yielding of the tension reinforcement. For such beams the steel stress f_{su} at the ultimate moment equals the yield stress f_y, and equation 5 gives the ultimate moment of the beam and is expressed as

$$M_u = A_s f_y\ (1 - k_2 k_u)d \quad \ldots\ldots(9)$$

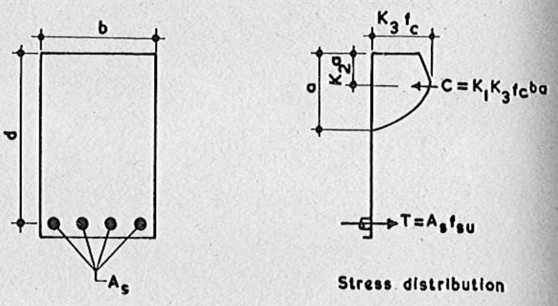

Fig 3 Stress distribution across the cross-section of a reinforced concrete beam at ultimate load

The procedure for calculating the ultimate moment of the all-lightweight aggregate concrete beams is given in *appendix A*. The details of the test results of beams are given in *Table* 4. The calculated ultimate moments (M_u) as obtained from equation 9 are also given. The data show that the experimental values M_{ex} and the theoretical values M_u are in good agreement.

The British and American codes of practice require a minimum load factor of 2·0 and 1·8, respectively [3,4]. The revised Indian Standard IS : 456-1964 specifies that the member should be designed to carry without failure a load equal to 1·5 DL + 2·2 LL. The load factor obtained for lightweight concrete beams ranged between 2·41 to 2·58, and that for dense concrete beams between 2·53 to 2·64. In other words, comparable load factors are obtained in both types of concrete beams.

TABLE 4 Results of the flexural test of beams

Designation of beam	*Compressive strength at 28 days, lb/in², First batch*	*Compressive strength at 28 days, lb/in², Second batch*	*Design moment, M_D, in lb×10³*	*Ultimate moment, due to applied load, in lb×10³*	*Dead load moment, in lb×10³*	*Total ultimate moment, M_{ex}, in lb×10³*	*Theoretical ultimate moment, M_u, in lb×10³*	$\frac{M_{ex}}{M_u}$	*Deflections at working loads, in: Central deflections, Theoretical, δ_1*	*Central deflections, Measured, δ*	*Deflection under load points, Theoretical, δ_2*	*Deflection under load points, Measured*	*Load factor* $\frac{M_{ex}}{M_D}$	*Span ÷ measured central deflection,* $\frac{L}{\delta}$
					All-lightweight aggregate concrete									
A_1	3066	3038	294·15	695	12·15	707·15	690	1·03	0·103	0·115	0·088	0·094	2·41	939
A_2	2954	3126	294·15	725	12·15	737·15	690	1·07	0·103	0·122	0·088	0·104	2·51	886
A_3	3000	2945	294·15	746	12·15	758·15	690	1·10	0·103	0·118	0·088	0·094	2·58	913
					Lightweight concrete containing sand									
B_1	3216	3234	295·39	746	13·39	759·39	719	1·05	0·090	0·092	0·077	0·081	2·57	1176
B_2	3252	3124	295·39	705	13·39	718·39	719	1·00	0·090	0·105	0·077	0·090	2·43	1029
B_3	3168	3103	295·39	746	13·39	759·39	719	1·05	0·090	0·097	0·077	0·082	2·57	1113
					Gravel concrete									
C_1	3118	3090	303·20	785	21·20	806·20	701	1·15	0·062	0·069	0·052	0·053	2·64	1565
C_2	3094	3266	303·20	746	21·20	767·20	701	1·09	0·062	0·076	0·052	0·059	2·53	1421
C_3	3070	3132	303·20	765	21·20	786·20	701	1·12	0·062	0·070	0·052	0·055	2·59	1541

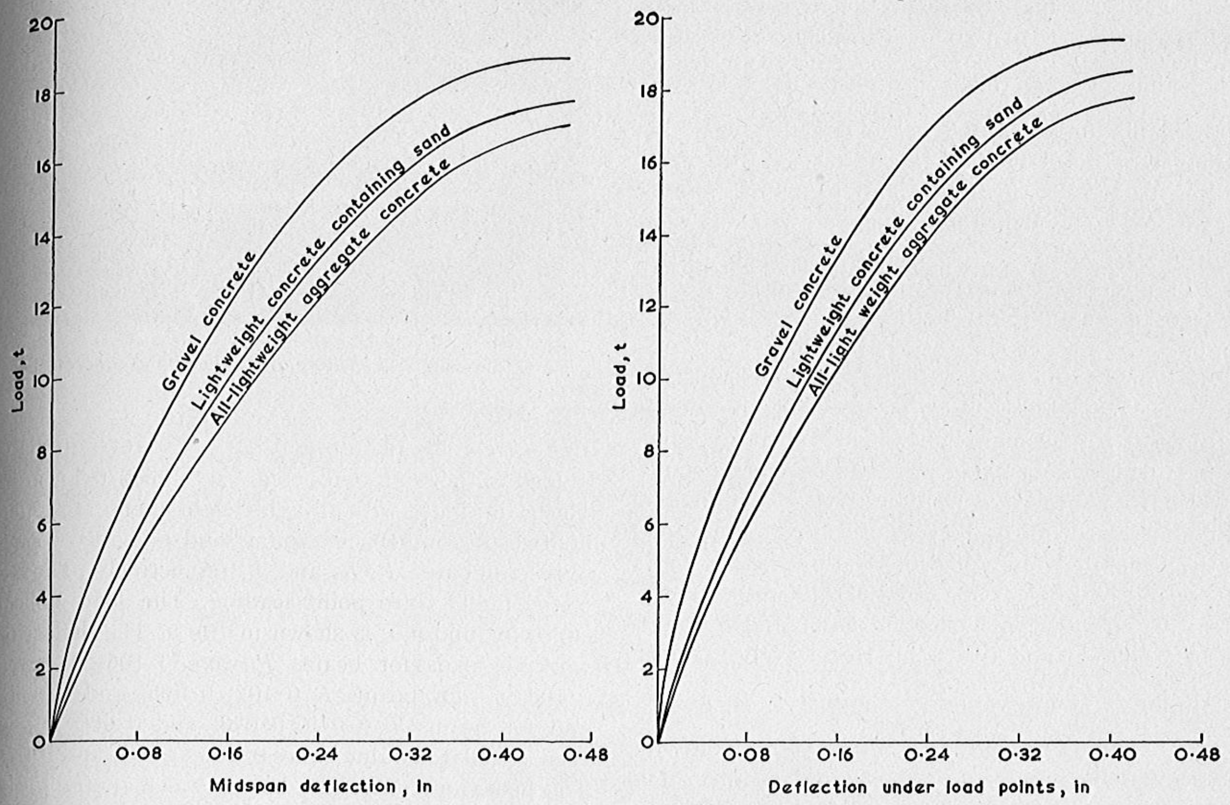

Fig 4 Typical load-deflection curves for beams tested in flexure. Left: midspan deflections. Right: deflections under load points

Similar results have also been reported elsewhere[8,9]. Both the lightweight and the dense concrete beams failed at loads higher than that specified by the IS : 456-1964. The data indicate that satisfactory load factors are obtained when the permissible stresses specified in the IS code of practice for conventional dense concrete are used in the design of lightweight concrete beams.

Deflections

Lightweight concrete members are known to deflect, more than dense concrete ones [9, 10, 11]. The central deflection δ_1 and the deflection under loading points δ_2 of a beam subjected to third point loading is calculated from the following equations :

$$\delta_1 = \frac{WL^3}{56 \cdot 4\ EI}$$

$$\delta_2 = \frac{WL^3}{64 \cdot 8\ EI}$$

where W is the total superimposed load, L the span, E the modulus of elasticity, and I the moment of inertia.

The above equations show that deflection is inversely proportional to the product EI. The load deflection curves for midspan and for loading points are shown in *Fig* 4. At the beginning when the concrete is not cracked, the entire section is effective, and I in the equations for deflection must be calculated considering the whole area of the concrete section. The contribution of steel to I is very small at this stage and may be neglected. Since the lightweight concrete beams are thinner in section and also the modulus of elasticity is low, their deflections at the beginning were much more than for the dense concrete beams. As the loading continued the differences in their deflections were reduced. (It must be noted that since the flexibility of reinforced concrete beams must be examined at working loads, the deflections in the early stages of the loading are not important). At working loads the concrete in the tension zone cracked and I was calculated on the basis of the cracked section. Since the steel has now considerable influence, it has to be taken into account. Also because the values of N and m of the lightweight concrete member are more, the concrete compression zone and steel reinforcement contributed more towards I.

The moment of inertia of all the beams is given in *Table* 3. It will be seen that the values of I for beams made with all-lightweight aggregate concrete and lightweight concrete containing sand are greater than that of the gravel concrete. The increased moment of inertia of the lightweight concrete beams appears to offset partly the adverse effects of the lower value of E for lightweight concretes. The EI values of lightweight concrete beams are, however, still lower than for dense concrete beams and consequently the lightweight concrete beams deflected more.

The observed and theoretical deflections at midspan and under the loading points of all the beams at working loads are given in *Table* 4. It will be seen that they show good agreement.

The ratio of the span to the measured central deflection $\frac{L}{\delta}$ at designed loads is shown in the last column of *Table* 4. $\frac{L}{\delta}$ values of A beams were 939, 886 and 913, for B beams 1176, 1029, 1113 and for C beams 1565, 1421 and 1541. The data indicate that though the flexural rigidity of the lightweight concrete beams was comparatively less than that of dense concrete beams, yet the values of $\frac{L}{\delta}$ for the former were much greater than the minimum acceptable value of 300.

It may be noticed that all the beams were designed according to the elastic theory and balanced sections were adopted. The lightweight concrete sections were comparatively thinner, but their $\frac{L}{\delta}$ values indicated that these sections are quite satisfactory.

For balanced sections, the ratio of the tensile reinforcement for lightweight concrete beams and gravel concrete beams was 1·24 and the ratio of their effective depth was about 0·85. Though lightweight concrete beams contained 24 per cent extra steel, its smaller effective depth led to increased deflection. If the section and steel reinforcement for both types of concrete beams are kept the same, the values of I for the three beams A, B, and C would be 2480, 2038, and 1360 in^4, respectively. The calculated values of deflection for A and B would then be 0·075 in and 0·072 in, respectively, against 0·062 in for C. The $\frac{L}{\delta}$ values for beams A, B, and C would be 1440, 1500, and 1740 respectively. Thus, by adopting the same sections, the flexural rigidity of a lightweight concrete beam can be brought at par with dense concrete beams, but the increased section of the lightweight concrete beam will result in increase of cement requirement and dead weight by about 25 per cent. It is, therefore, left to the designer to select either a balanced section based on the value of m for lightweight concrete or an under-reinforced section based on the value of m for dense concrete.

Beams for the shear tests

The shear beams were 9 in wide by 13 in deep (effective) and 10·5 ft long, and were reinforced with three $\frac{3}{4}$ in dia and one $\frac{1}{2}$ in dia plain round bars. Bars of $\frac{3}{8}$ in dia were welded to the main reinforcement to keep it in position. The beams made with all-lightweight aggregate concrete, lightweight concrete containing sand, and gravel concrete were designated P, R, and T, respectively. They were tested under third point loading. The load deflection curve for midspan is shown in *Fig* 5. The deflection at working loads for beams P were 0·106, 0·103, and 0·108 in; for beams R 0·102, 0·093, and 0·095 in; and for beams T 0·079, 0·073, and 0·080 in; while their calculated values were 0·103, 0·096, and 0·078 in. The behaviour of all the beams was essentially elastic until diagonal tension cracks appeared. First, flexural cracks appeared within the range of constant moment. As load increased, a diagonal tension crack appeared between the support and the nearest load point. A similar crack formed on the other half of the beam. These cracks developed more rapidly than the vertical cracks in the region of pure flexure. The diagonal tension crack extended almost horizontally under the load point and invaded the pure moment region of the beam. Because of the sudden occurrence of the failure crack at one end of the beam, the extensive development of the diagonal crack at the other end was prevented (*Fig* 6). In some cases, the splitting of the beam along a horizontal plane was noticed at the level of the reinforcement. The splitting phenomenon occurred after the formation of the failure crack and as such had no significant effect on the ultimate strength of the beam in shear.

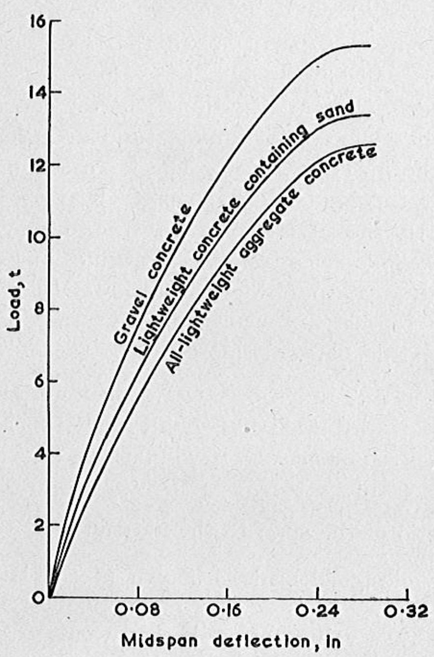

Fig 5 Typical load deflection curves for beams tested in shear

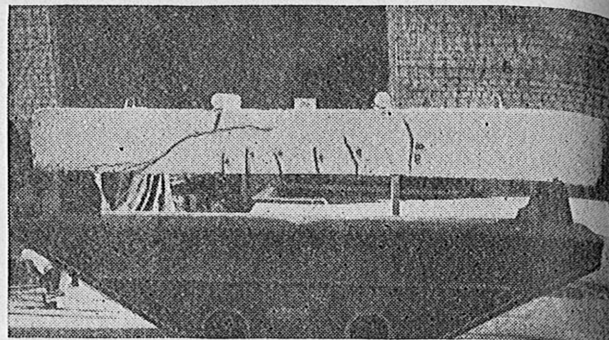

Fig 6 Showing shear failure of a lightweight concrete beam

Ultimate shear strength

The failure of a beam in shear is due to the principal stresses leading to diagonal cracking. The load causing failure is, therefore, referred to as a diagonal cracking load. The ultimate shear strength of a reinforced concrete beam depends upon its shear span [10,12]. The diagonal cracking load decreases slightly as the shear span increases, in other words, the bending moment also has an influence on the diagonal cracking. The shear strength of the concrete beam should, therefore, be calculated on the basis of shear force at diagonal cracking corresponding to a maximum bending moment on the beam equal to the calculated ultimate resistance moment.

Fig 7 shows the relation between the ratio of the moment at diagonal cracking to the calculated ultimate

TABLE 5 Results of the shear tests of beams

Designation of beam	Compressive strength at 28 days, lb/in^2 First batch	Compressive strength at 28 days, lb/in^2 Second batch	Ultimate moment due to applied load, in $lb \times 10^3$	Dead load moment, in $lb \times 10^3$	Total ultimate moment, M_s, in $lb \times 10^3$	Theoretical ultimate moment, M_u, in $lb \times 10^3$	$\frac{M_s}{M_u}$	Diagonal cracking load, tons Q	Diagonal cracking load, tons Q_u	Shear strength, $q = \frac{Q_u}{bd}$, lb/in^2	Deflection at working load, in Theoretical	Deflection at working load, in Measured
					All-lightweight aggregate concrete							
P_1	3024	2940	495	12·15	507·15	690	0·735	6·31	5·81	112	0·103	0·106
P_2	3056	2992	485	12·15	497·15	690	0·722	6·185	5·75	110	0·103	0·103
P_3	3078	3112	524	12·15	536·15	690	0·778	6·685	6·30	120	0·103	0·108
					Lightweight concrete containing sand							
R_1	3174	3244	564	13·30	577·30	690	0·835	7·205	6·92	132	0·096	0·102
R_2	3300	3252	521	13·30	534·30	690	0·774	6·705	6·36	121	0·096	0·093
R_3	2980	3168	555	13·30	568·30	690	0·825	7·080	6·75	129	0·096	0·095
					Gravel concrete							
T_1	3160	3294	645	18·2	663·20	6900	0·96	8·28	8·20	156	0·078	0·079
T_2	3266	3360	604	18·2	622·20	6900	0·90	7·78	7·55	144	0·078	0·073
T_3	3248	3104	585	18·2	603·20	6900	0·875	7·51	7·24	139	0·078	0·080

resistance moment, $\frac{M_s}{M_u}$, and the ratio of the actual diagonal cracking load to the diagonal cracking load corresponding to $M_s = M_u, \frac{Q}{Q_u}$ [13]. For a beam in which M_s is equal to M_u, the shear force which would cause diagonal cracking is given by $Q_u = qbd$.

where q is the shear strength.

When M_s is less than M_u, the shear force to cause diagonal cracking is somewhat greater. The appropriate value of Q_u, and hence q, for a beam can be obtained from the value of $\frac{M_s}{M_u}$. The results of the shear tests of the beams are given in *Table* 5. It will be seen that the values of the shear strength for beams P are 75·5 to 82·4 per cent, and for beams R 82·9 to 90·5 per cent, of that for beams T. This is in agreement with the findings of other workers [8, 12, 14].

In view of the results obtained, it is necessary to reduce the permissible shear stress for lightweight concrete members by 25 per cent compared with gravel concrete members. With the exception of slabs, lintels, and similar members in which shear stresses are generally low, reinforced lightweight concrete flexural members must contain shear reinforcement in accordance with the IS code of practice.

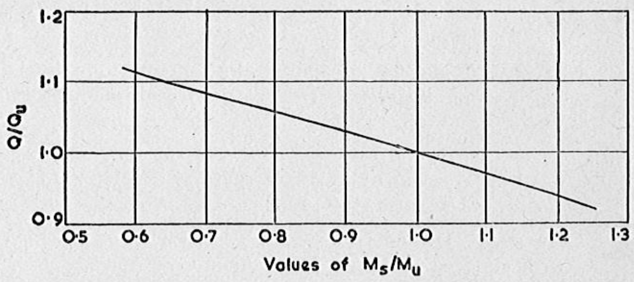

Fig 7 Showing the relationship between M_s/M_u and Q/Q_u

Conclusions

The conclusions drawn from the investigations are :

1 Lightweight concrete beams can be designed according to the IS code of practice.

2 Similar load factors are obtained both in lightweight concrete and gravel concrete beams under flexure.

3 The flexural rigidity of lightweight concrete beams is comparatively less than that of dense concrete beams but values of $\frac{L}{\delta}$ for the former are much greater than the minimum acceptable value of 300.

4 It is left to the designer to select either a balanced section based on the value of m for lightweight concrete or an under-reinforced section based on the value of m for dense concrete.

5 The shear strength of lightweight concrete beams is 75·5 to 82·4 per cent of that for dense concrete. The use of sand in lightweight concrete increases the values to 82·9 to 90·5 per cent.

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Appendix I Design of all-lightweight aggregate flexural beams for the flexural test

Mix composition 1 : 1·5 : 3 (by volume)

Permissible compressive stress in concrete, c = 1,000 lb/in²

Permissible tensile stress in steel, t = 18,000 lb/in²

Modular ratio, m = 15·9

Depth coefficient of neutral axis, N = 0·47

Coefficient of lever arm, J = 0·843

Resistance moment factor, Q = 198

Superimposed load, P = 7 tons

Dead weight of the beam, w = 100 lb/rft

Bending moment due to superimposed load $= \frac{PL}{6}$

$$= \frac{7 \times 9 \times 2240 \times 12}{6}$$

$= 2{,}82{,}000$ in lb

Dead weight moment $= \frac{wL^2}{8}$

$$= \frac{100 \times 9 \times 9}{8} \times 12$$

$= 12{,}150$ in lb

Designed moment $= 2{,}94{,}150$ lb

Assuming width of the beam, b = 9 in

Effective depth, d $= \sqrt{\frac{294150}{198 \times 9}}$

$= 12 \cdot 8$ in

Adopt 13 in as the effective depth and 14·50 in as the overall depth

Tension reinforcement, A_s $= \frac{294150}{18000 \times 0 \cdot 843 \times 13}$

$= 1 \cdot 49$ in²

Provide three $\frac{3}{4}$ in dia and one $\frac{1}{2}$ in dia bars

Bond stress $= \frac{3 \cdot 5 \times 2240 + 450}{0 \cdot 843 \times 13\,(3 \times 2.36 + 1 \cdot 57)} = 87 \cdot 2$ lb/in²

Shear stress $= \frac{3 \cdot 5 \times 2240 + 450}{9 \times 0 \cdot 843 \times 13} = 83 \cdot 7$ lb/in²

Since the beam is to be tested to destruction, it should contain sufficient shear reinforcement.

Provide $\frac{3}{8}$ in dia two-legged stirrups at 4·5 in o.c. in the shear span and 12 in o.c. in the region of pure bending moment.

Ultimate moment

$$k_1 = 0 \cdot 94 - \frac{f_c}{26000} = 0 \cdot 854$$

$$k_2 = 0 \cdot 50 - \frac{f_c}{80000} = 0 \cdot 472$$

$$k_3 = \frac{3900 + 0 \cdot 35 f_c}{3000 + 0 \cdot 82 f_c - \frac{f_c^2}{26000}} = 1 \cdot 009$$

$$k_1\, k_3\, f_c\, b\, a = A_s\, f_y$$

$$a = \frac{1 \cdot 516 \times 40000}{0 \cdot 854 \times 1 \cdot 009 \times 2250 \times 9} = 3 \cdot 49 \text{ in}$$

$$k_u = \frac{a}{d} = 0 \cdot 268$$

$$M_u = A_s f_y (1 - k_2 k_u)\, d$$

$$= 1 \cdot 516 \times 40000\,(1 - 0 \cdot 472 \times 0 \cdot 268) \times 13$$

$$= 69{,}000 \text{ in lb}$$

"....in wet weather drivers travel further apart and deceleration waves are never amplified ; it is not necessary to control the expressway in wet weather. This suggests that the cheapest method of expressway control may be to spray the road with water during peak periods !"

A. J. Miller in "Endeavour"

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