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Design tables for folded plates

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These tables are offered as a basis for making a choice of three commonly employed shapes of folded plates and fixing dimensions for a preliminary design. The logic behind the tables, explained in detail in the article, is that once the stress resultants for a particular span are known, they can be found for another span.

Folded plates are now extensively used to roof large column-free areas and have been found ideally suitable for industrial structures. Architects and engineers in India have lately become interested in this form of construction, various papers published by the Central Building Research Institute having contributed to some extent in popularising it.[1,2,3,4,5] This paper is another step in this direction. It aims at providing ready-made design tables of stress resultants for a large number of folded plates. Commonly employed shapes of folded plates have been chosen and transverse moments and longitudinal stresses for different spans have been presented.

Method of analysis

The detailed study of the various methods of folded plate analysis has revealed that the Simpson method[6] is more flexible than the Whitney method[7] in arriving at the stress resultants for various spans with a constant cross-section. The procedure which forms the basis for the design tables is enumerated in the following paragraphs. Readers are referred to the original papers of Simpson and Whitney for a better understanding of the procedure.

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The necessary changes to be incorporated in the Simpson and Whitney methods for the changes in span are enumerated below. The Simpson method is explained in somewhat greater detail than the Whitney method as the former is recommended for preparing similar tables.

Simpson's method

1. The external load is divided into components parallel and perpendicular to the plate on which it acts. A strip of unit width is then analysed as a continuous beam in the transverse direction, the ridges being assumed unyielding. Thus, transverse moments $m^o{}_k$ and plate loads $R^o{}_k$ are obtained. This step is referred to as the 'no rotation' solution.
2. From the plate loads $R^o{}_k$, longitudinal stresses $\frac{R^o{}_k L^2}{8\,Z_k}$ and subsequent 'distributed' stresses $A_k\,L^2$ are obtained.
3. To account for the transverse relative displacements of the joints each plate is rotated, in turn, arbitrarily, and transverse moments M_k' and longitudinal stresses $A_k'\,L^2$ for every case of 'rotation' are obtained. These calculations are referred to as the 'rotation' solutions.

NOTATION

L	= span of folded plate between diaphragms	$\delta^o{}_k$	= in-plane deflection of plate k for the 'no rotation' solution
t_k	= thickness of plate k	δ_k'	= in-plane deflection of plate k for one of the 'rotation' solutions
w_k	= width of plate k	Δ_k	= final in-plane deflection of plate k
γ_k	= angle between two plates at k measured from the inferior plate in the clockwise direction towards the superior plate	ψ_k	= final rotation of plate k
Z_k	= section modulus of plate k	$\psi^o{}_k$	= arbitrary rotation of plate k
$R^o{}_k$	= load in plane of plate k for the 'no rotation' solution	K	= arbitrary constant which is the ratio of the final rotation to the arbitrary rotation of the plate.
$m^o{}_k$	= transverse moments at joint k for the 'no rotation' solution	A_k, A_k', B_k', B_k, etc	= constants depending upon the cross-sectional properties
M_k'	= transverse moment at joint k for one of the 'rotation' solutions		

4. From the stresses at the top and bottom of each plate, in-plane deflections of the plates are obtained for the 'no rotation' solution $[\delta^o_k = B_k L^4]$ and the 'rotation' solution $[\delta_k' = B_k' L^4]$.

5. Final in-plane deflection of the plate k can be expressed as follows:

$$\Delta_k = \delta^o_k + K_1\,\delta_k' + K_2\,\delta_k'' + K_3\,\delta_k''' + \dots$$
$$= B_k L^4 + K_1 B_k' L^4 + K_2 B_k'' L^4 + K_3 B_k''' L^4 + \dots\dots\dots \quad (1)$$

Here, K_1, K_2, etc are the arbitrary constants for the plates 1, 2, etc and are equal to the ratio of the actual plate rotation to the arbitrary rotation, *i.e.*, $\psi_k = K\psi^o_k$.

6. Knowing the in-plane deflections, rotations of the plates ψ_k are obtained from the formula based on the deformed geometry of the cross-section.

$$\psi_k = \frac{1}{w_k}\left[\Delta_k(\cot\gamma_k + \cot\gamma_{k-1}) - \frac{\Delta_{k+1}}{\sin\gamma_k} - \frac{\Delta_{k-1}}{\sin\gamma_{k-1}}\right] \quad \dots\dots\dots \quad (2)$$

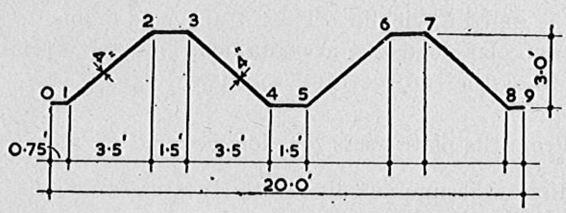
$$= \frac{L^4}{w_k}\left[\begin{array}{l}\text{Terms in } k_1, k_2, \text{ etc and} \\ \text{a numerical term}\end{array}\right] \quad \dots\dots\dots \quad (3)$$

7. But $\psi_k = K\psi^o_k$ and the arbitrary rotation is proportional to $\frac{w_k}{t^3_k}$, hence, taking the factor $\frac{L^4}{w_k}$ of the equation (3) to the left-hand side, we get

$$\frac{w_k^2}{t_k^3 L^4} = \left[\begin{array}{l}\text{Terms in } k_1, k_2, \text{ etc plus} \\ \text{a numerical term origina-} \\ \text{ting from the 'no rotation'} \\ \text{solution}\end{array}\right] \quad \dots\dots \quad (4)$$

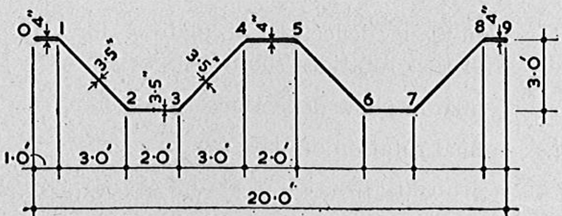
8. Simultaneous equations resulting from equation (4) are solved for K_1, K_2, K_3, etc. Knowing these arbitrary

Trough-shaped folded plates



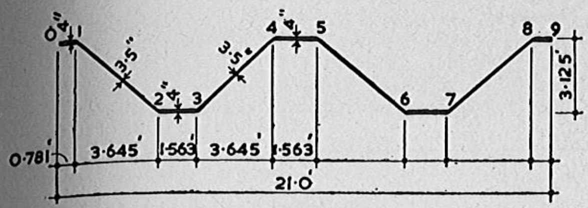
Live load : 10 lb/ft² of surface area

Span, ft \ Joint	0	1	2	3	4
		Longitudinal stresses, lb/in²			
60	+ 670·00	+ 800·90	− 762·10	− 755·10	+ 730·90
55	+ 563·00	+ 672·90	− 640·40	− 634·60	+ 614·10
50	+ 480·18	+ 573·97	− 546·15	− 541·24	+ 523·82
		Transverse moments, lb ft/ft			
60	—	+ 16·88	+ 923·80	+ 898·60	− 161·20
55	—	+ 16·88	+ 923·05	+ 892·62	− 197·37
50	—	+ 16·88	+ 920·00	+ 883·00	− 240·00



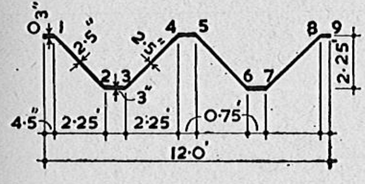
Live load : 15 lb/ft² of surface area

Span, ft \ Joint	0	1	2	3	4
		Longitudinal stresses, lb/in²			
65	− 1118·68	− 822·87	+ 874·75	+ 904·00	− 863·99
60	− 946·82	− 705·35	+ 747·92	+ 769·37	− 734·51
55	− 784·00	− 598·61	+ 632·25	+ 643·39	− 615·38
50	− 638·36	− 499·61	+ 525·65	+ 529·18	− 507·09
		Transverse moments, lb ft/ft			
65	—	+ 32·50	+ 900·85	+ 847·98	− 256·75
60	—	+ 32·50	+ 900·23	+ 845·30	− 266·22
55	—	+ 32·50	+ 898·91	+ 841·93	− 274·90
50	—	+ 32·50	+ 897·60	+ 838·66	− 280·96



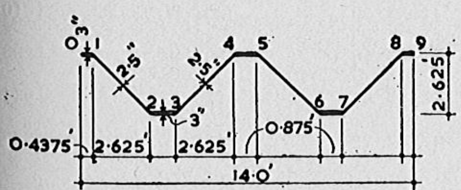
Live load : 15 lb/ft² of surface area.

Span, ft \ Joint	0	1	2	3	4
Longitudinal stresses, lb in²					
60	− 1076·74	− 728·75	+ 738·12	+ 791·11	− 756·81
55	− 904·76	− 612·35	+ 620·23	+ 664·75	− 635·93
50	− 747·74	− 506·08	+ 512·58	+ 549·38	− 525·56
45	− 534·60	− 431·00	+ 422·50	+ 443·70	− 423·00
Transverse moments, lb ft/ft					
60	—	+ 19·84	+ 985·58	+ 953·07	− 271·23
55	—	+ 19·84	+ 982·59	+ 948·20	− 284·06
50	—	+ 19·84	+ 979·59	+ 943·35	− 296·88
45	—	+ 19·84	+ 976·59	+ 938·50	− 309·70



Live load: 10 lb/ft² of surface area

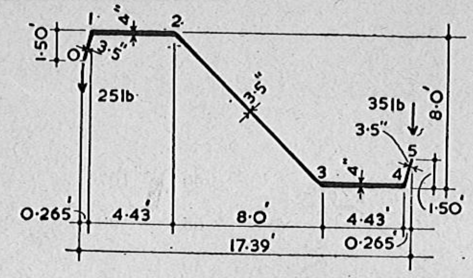
Span, ft \ Joint	0	1	2	3	4
Longitudinal stresses, lb/in²					
55	− 654·35	− 1048·87	+ 966·30	+ 959·16	− 920·23
50	− 540·78	− 866·84	+ 798·60	+ 792·70	− 760·52
45	− 438·04	− 702·14	+ 646·86	+ 642·09	− 616·02
40	− 346·10	− 554·77	+ 511·10	+ 507·33	− 486·73
Transverse moments, lb ft/ft					
55	—	+ 4·25	+ 205·55	+ 202·94	− 18·40
50	—	+ 4·25	+ 205·03	+ 201·66	− 25·01
45	—	+ 4·25	+ 204·51	+ 200·38	− 31·62
40	—	+ 4·25	+ 204·00	+ 199·10	− 38·23



Live load: 10 lb/ft² of surface area

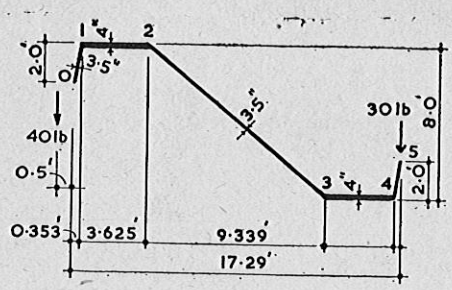
Span, ft \ Joint	0	1	2	3	4
Longitudinal stresses, lb/in²					
60	− 839·05	− 1067·51	+ 1007·44	+ 988·75	− 947·96
55	− 705·04	− 897·01	+ 846·53	+ 830·83	− 796·55
50	− 582·68	− 741·33	+ 699·61	+ 686·63	− 658·30
45	− 471·96	− 600·48	+ 566·68	+ 556·17	− 533·23
Transverse moments, lb ft/ft					
60	—	+ 5·79	+ 284·52	+ 279·38	− 38·08
55	—	+ 5·79	+ 284·21	+ 278·11	− 45·94
50	—	+ 5·79	+ 283·90	+ 276·84	− 53·80
45	—	+ 5·79	+ 283·59	+ 275·57	− 61·66

Northlight folded plates



Live load: 15 lb/ft² of surface area

Span, ft \ Joint	0	1	2	3	4	5
			Longitudinal stresses, lb/in²			
60	+ 1467·80	+ 460·96	− 973·67	+ 980·49	− 472·02	− 1492·60
50	+ 1061·16	+ 301·71	− 663·08	+ 672·50	− 308·00	− 1081·36
45	+ 893·45	+ 229·48	− 534·60	+ 538·00	− 233·45	− 912·22
40	+ 705·94	+ 181·32	− 422·40	+ 425·09	− 184·45	− 720·77
			Transverse moments, lb ft/ft			
60	—	+ 18·45	+ 682·19	+ 728·52	+ 21·09	—
50	—	+ 18·45	+ 657·83	+ 702·42	+ 21·09	—
45	—	+ 18·45	+ 645·65	+ 689·42	+ 21·09	—
40	—	+ 18·45	+ 633·47	+ 676·42	+ 21·09	—



Live load: 15 lb/ft² of surface area

Span, ft \ Joint	0	1	2	3	4	5
			Longitudinal stresses, lb/in²			
60	+ 1640·00	+ 320·00	− 987·00	+ 984·50	− 321·80	− 1610·00
55	+ 1321·52	+ 301·14	− 844·45	+ 843·48	− 305·04	− 1299·20
45	+ 846·81	+ 223·17	− 575·40	+ 575·51	− 227·38	− 834·86
40	+ 669·08	+ 176·33	− 454·64	+ 454·72	− 179·66	− 659·64
			Transverse moments, lb ft/ft			
60	—	+ 55·10	+ 242·10	+ 187·50	+ 31·60	—
55	—	+ 55·10	+ 250·59	+ 196·44	+ 31·60	—
45	—	+ 55·10	+ 279·23	+ 227·48	+ 31·60	—
40	—	+ 55·10	+ 301·79	+ 252·73	+ 31·60	—

constants, final transverse moments at midspan can be obtained as follows :

$$M_k = m^o{}_k + \Sigma K\, M_k' \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

The final longitudinal stresses at midspan can be obtained as

$$A_k L^2 + \Sigma\, K A_k' L^2$$
$$= L^2 [A_k + \Sigma K A_k'] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

Equations (5) and (6) reveal that for a given cross-section, the transverse moments depend upon the values of the arbitrary constants K, whereas the longitudinal stresses depend upon the arbitrary constants K as well as the span of the folded plate.

Thus, if a problem is solved for a span L and for a certain cross-section and then, if the stress-resultants are required for some other span L, the only change to be incorporated is in equation (4) of step 7, all other steps remaining unchanged, *i.e.*, the left-hand side of equation (4) must be multiplied by a factor $\left[\frac{L^4}{L'^4}\right]$

The Whitney method

The only revision necessary in the problem when it is solved by the Whitney method is in the final set of simultaneous equations. These equations are based on the condition that the total change in the angle at any joint is zero. The change in angle results from

- the rotations of the plates at the joints caused due to 'slab action' of the external loads
- the rotations of the plates at the joints caused due to the transverse moments
- the angle change caused due to the in-plane deflections of the plates.

Of the three quantities, the first two remain constant irrespective of the span. Thus, the only change occurs in the third quantity. Unlike in the Simpson method, the unknowns in the final set of equations are transverse moments. As such, they can be obtained very easily for any span. But the calculations for longitudinal stresses involve evaluation of intermediate quantities like plate moments and longitudinal shears.

After solving a large number of folded plates, it has been observed that the term on the left-hand side of equation (4) in the Simpson method is either very small or fairly large compared to its corresponding term on the right-hand side. As such, when a problem is solved taking into account the term on the left-hand side, any subsequent multiplication of that term with $\left[\frac{L^4}{L'^4}\right]$ would make very little change in the magnitude, if the quantity is initially large or the entire quantity on the left-hand side becomes negligible, if it is initially small. Thus one set of K values will hold good for a certain range of spans, *i.e.*, the transverse moments in that range remain constant irrespective of the span and the longitudinal stresses are directly proportional to L^2 for all practical purposes. In fact, these are the conclusions drawn in the simple beam theory. It is also well known that the beam theory has limited application to folded plates. There has, therefore, to be a limitation on the value of the term $\left[\frac{L'^4 - L^4}{L'^4}\right]$ so that the above conclusion may remain valid.

The entire problem thus centres on the sensitivity of the left-hand side quantity of equation (4) for the Simpson method, and the relative magnitude of the angle change due to the plate deflections compared to the first two quantities in the Whitney method. In one of the problems, it was noticed that even the total omission of the quantity on the left-hand side in equation (4) made very little difference in the final stress resultants. However, the transverse moments are invariably found to be more sensitive to the K values than the longitudinal stresses.

In the design tables, for certain shapes, simultaneous equations were solved for every span. In certain cases the transverse moments were obtained for every span but the longitudinal stresses were deducted proportional to L^2. However, the procedure discussed in this paper enables us to work out stress resultants for any range of spans by adopting the following steps :

1. A problem is solved by the Simpson method for a particular span, say 50 ft.
2. This solution enables us to find out stress resultants within a range of 45 ft to 55 ft. The longitudinal stresses are assumed proportional to L^2 and the transverse moments constant within the range of $\pm$ 5 ft.
3. The final set of simultaneous equations arising from equation (4) are solved again for a span, say, 65 ft in place of 50 ft. This amounts to a change in the left-hand side term of the order of $\left[\frac{L'^4 - L^4}{L'^4}\right]$. This solution furnishes stress resultants for a range of 60 ft to 70 ft.

Thus, the variation of the stress resultants over a wide range of 45 ft to 70 ft is known sufficiently accurately to proceed with the preliminary design or to choose the dimensions of the folded plate.

All these explanations are given here to indicate the logic employed in the design tables. As far as the designer is concerned, he may pick any one of the shapes and a suitable corresponding span from the tables.

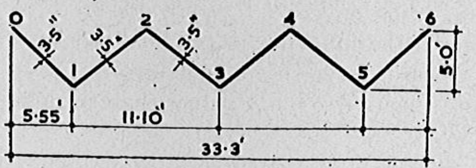
Design tables

The tables given herein pertain to three shapes which are most frequently used. Structural depths have been chosen so that the final stresses in the given range of spans are somewhere around the permissible stresses. A live load of 15 lb/ft² of surface area is assumed, unless specified in the problem. The span variation is limited to the range of 40 ft to 70 ft as prestressing would be generally necessary for spans exceeding 70 ft, and folded plates for less than 40 ft are generally uneconomical.

Conclusions

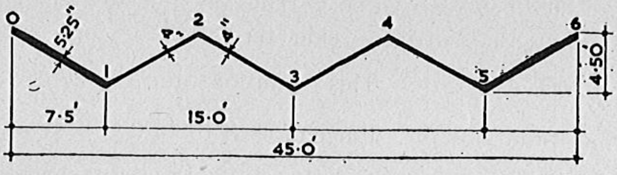
1. The design tables offer a basis for comparing economical proportioning of cross-sections for a given span. In fact, this is the problem that faces a designer when he

V-shaped folded plates



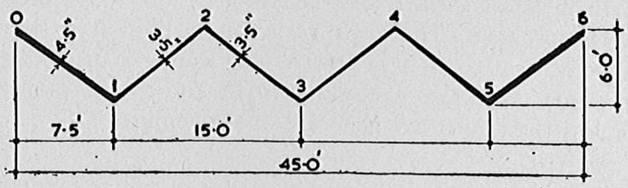
Live load: 10 lb/ft² of surface area

Span, ft \ Joint	0	1	2	3
		Longitudinal stresses, lb/in²		
65	− 870·55	+ 878·83	+ 826·05	+ 764·98
60	− 741·77	+ 748·82	− 703·85	+ 651·82
55	− 623·29	+ 629·22	− 591·43	+ 547·71
		Transverse moments, lb ft/ft		
65	—	+ 1120·43	− 68·40	+ 317·90
60	—	+ 1120·43	− 77·00	+ 342·20
55	—	+ 1120·43	− 85·61	+ 366·50



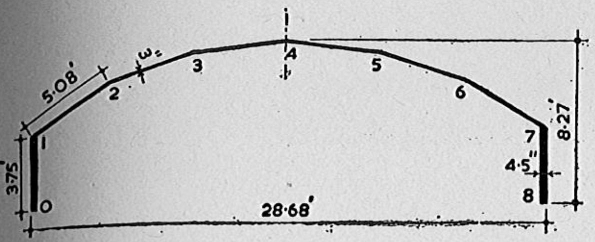
Live load: 15 lb/ft² of surface area

Span, ft \ Joint	0	1	2	3
		Longitudinal stresses, lb/in²		
65	− 928·00	+ 973·00	− 935·00	+ 842·00
60	− 792·00	+ 829·00	− 797·00	+ 715·00
55	− 665·00	+ 697·00	− 670·00	+ 600·00
		Transverse moments, lb ft/ft		
65	—	+ 2644·60	− 339·30	+ 832·20
60	—	+ 2644·60	− 328·30	+ 793·10
55	—	+ 2644·60	− 320·20	+ 761·90



Live load : 15 lb/ft² of surface area.

Span, ft \ Joint	0	1	2	3
		Longitudinal stresses, lb/in²		
70	− 834·96	+ 874·32	− 839·49	+ 754·15
65	− 719·94	+ 753·88	− 723·85	+ 650·26
60	− 613·44	+ 642·36	− 616·77	+ 554·07
		Transverse moment lb ft/ft		
70	—	+ 2566·15	− 299·68	+ 743·77
65	—	+ 2566·15	− 294·57	+ 724·56
60	—	+ 2566·15	− 290·51	+ 709·30



Live load : 25 lb/ft² of horizontal projection.

Span, ft \ Joint	0	1	2	3	4
			Longitudinal stresses, lb/in²		
70	+ 859·33	+ 188·33	− 97·14	− 387·48	− 400·75
65	+ 742·47	+ 164·63	− 92·09	− 333·34	− 336·59
60	+ 632·64	+ 140·28	− 78·47	− 284·03	− 286·80
			Transverse moments, lb ft		
70	—	—	− 212·80	+ 206·35	+ 436·83
65	—	—	− 215·71	+ 198·92	+ 426·99
60	—	—	− 218·62	+ 191·49	+ 417·15

is fixing the preliminary dimensions. Since the quantity of mild steel in any proposal depends upon the values of longitudinal stresses and transverse moments, the tables present very useful information for purposes of preliminary design.

2. On comparing the various shapes for a constant span, it is observed that the trough shape is more efficient than the V. The efficient structural depth for a trough type folded plate varies between 1/15 to 1/18 of the span, whereas for the V-shape, a height of 1/10 to 1/12 of the span is necessary. For northlight folded plates the structural depth (in the real sense) being inclined, a vertical depth of $\frac{1}{6}$ to $\frac{1}{8}$ of the span is required.

3. Transverse moments assume a proportionately high value at joints 2 and 3 for a trough shape. For a constant trough type of cross-section, as the span is reduced the transverse moments at joints 2 and 3 increase, whereas the transverse moment at joint 4 decreases. The largest value of the transverse moments for a V-shape is the free cantilever moment at the joint 1.

4. Stresses of very high order are observed at the free edges of the northlight folded plates. However, the stress level in the interior is fairly low.

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