Reprinted from the "Proceedings of the Indian Academy of Sciences," Vol. LI, 1960

THERMAL DIFFUSIVITY OF A ROOF SLAB FROM *IN SITU* TEMPERATURE MEASUREMENTS

BY

B. C. RAYCHAUDHURI

Reprinted from "The Proceedings of the Indian Academy of Sciences",
Vol. LI, No. 5, Sec. A, 1960

THERMAL DIFFUSIVITY OF A ROOF SLAB FROM *IN SITU* TEMPERATURE MEASUREMENTS

BY B. C. RAYCHAUDHURI
(*Central Building Research Institute, Roorkee*)

Received September 24, 1959
(Communicated by T. N. Seshadri, F.A.SC.)

INTRODUCTION

THE flow of heat through a roof slab is periodic, on account of the solar insolation, which imposes a diurnal variation of temperature on its exposed surface. The exact periodicity can be vitiated by the small difference that may occur between the initial and final temperatures for the 24-hour interval: but this difference is usually negligible, especially when the day forms part of a stable synoptic regime. The significant thermal property of the roof material is not so much its thermal conductivity (K), as its thermal diffusivity, α given by the relation

$$\alpha = \frac{K}{\rho C_p}$$

where ρ and C_p are its density and specific heat respectively.

In common practice, α is calculated from the experimental values of K, ρ, and C_p measured under steady state conditions for oven-dry specimens. The temperature dependence of these constants may be a second order effect only; but it is definitely known that they vary with the moisture content of the material. Since the air in the pores of a moist specimen is replaced by the more conducting water both in its liquid and vapour phases, not only the effective ρ, and C_p but K also alters. A vapour pressure gradient is set up due to temperature differences; and the consequent vapour movement inside gives rise to an additional heat transport, both latent and sensible, in parallel to the conduction flow. This reduces the thermal resistance or increases the apparent conductivity of the material.

Considerable errors are therefore likely to be introduced in the analysis of heat transfer through building sections if α is calculated from the conventionally determined values of K, ρ and C_p. The correct approach would be to evolve a method of determining α of the material directly under the existing conditions of moisture.

An attempt is made in this paper to obtain a value for α for a roof slab from *in situ* temperature measurements of its outside and inside surfaces and of the ambient air inside the room below.

EXPERIMENTAL

Temperature data were available already, for the living rooms of about 40 dwelling houses put up for the Low Cost Housing Exhibition at Delhi (Raychaudhuri, 1957). Thermocouples were set up at a number of points in the experimental rooms and hourly temperatures were recorded continuously for a number of days, to an accuracy of $\pm 0\cdot1°$ F.

The data used in this paper were that obtained for one of the rooms, kept completely closed throughout a clear day (22nd April 1955) in the middle of a stable stretch of weather conditions. The roof of the room was a $4\frac{1}{2}''$ R.C.C. slab. The room was $10'\,6'' \times 12' \times 9'\,9''$ in dimensions and built with 9″ brick walls in cement mortar, plastered on both sides.

Curves for the observed surface temperatures of the roof (outside) the ceiling (inside) and the ambient air temperature at the centre of the room, on the chosen day are given in Fig. 1.

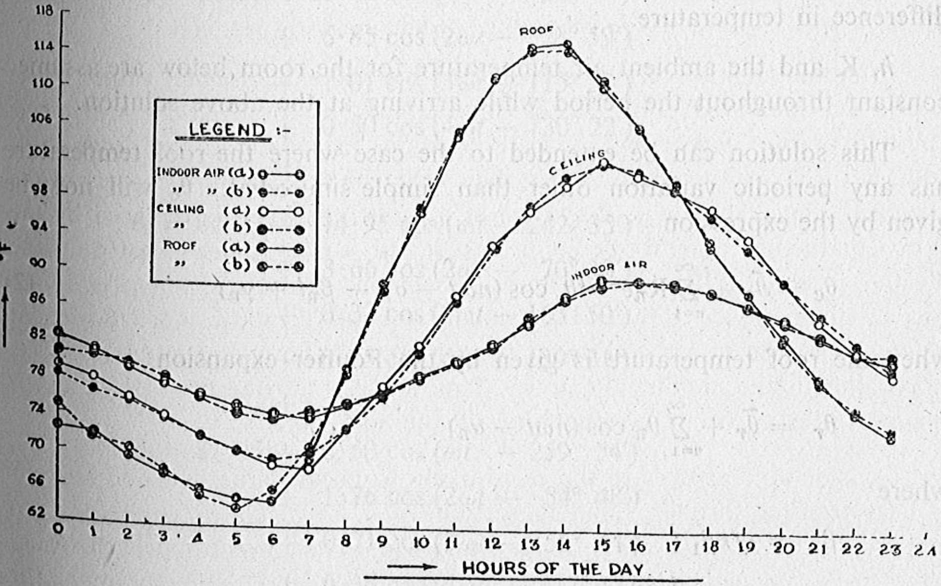

FIG. 1. Time-temperature curves for roof ceiling and indoor ambient air, (*a*) As observed, (*b*) As synthesised from the Fourier components.

THEORETICAL

Houghten (1932) and his co-workers have given a solution for the ceiling temperature, θ_c consequent on a unidirectional transverse flow of heat from the roof surface where the temperature varies sinusodially, with an amplitude θ_1.

The solution is given below in a slightly altered form

$$\theta_c = R_1 e^{-\beta_1 l}\theta_1 \sin(\omega t - \beta_1 l + \gamma_1) \quad (1)$$

where l is the thickness of the roof slab;

$$\beta_1 = \sqrt{\frac{\omega}{2a}}, \quad \left(\omega = \frac{2\pi}{T} = 15°/\text{hr.}\right)$$

$$R_1 = \frac{2\sqrt{2}}{\sqrt{2 + 2f_1 + f_1^2}};$$

$$\gamma_1 = \tan^{-1}\frac{f_1}{2 + f_1};$$

$$f_1 = \frac{h}{K\beta_1},$$

and h is the appropriate film transfer coefficient, equal to the ratio between the rate of heat transfer per unit area of the ceiling and the ceiling-to-air difference in temperature.

h, K and the ambient air temperature for the room below are assumed constant throughout the period while arriving at the above solution.

This solution can be extended to the case where the roof temperature has any periodic variation other than simple sinusodial. θ_c will now be given by the expression

$$\theta_c = \overline{\theta}_c + \sum_{n=1}^{\infty} R_n e^{-\beta_n l}\theta_n \cos(n\omega t - \sigma_n - \beta_n l + \gamma_n) \quad (2)$$

when the roof temperature is given by the Fourier expansion

$$\theta_r = \overline{\theta}_r + \sum_{n=1}^{\infty} \theta_n \cos(n\omega t - \sigma_n)$$

where

$$\beta_n = \sqrt{n}\beta_1;$$

$$R_n = \frac{2\sqrt{2}}{\sqrt{2 + 2f_n + f_n^2}};$$

$$\gamma_n = \tan^{-1} \frac{f_n}{2 + f_n}$$

and

$$f_n = \frac{f_1}{\sqrt{n}}.$$

$\overline{\theta}_r$ and $\overline{\theta}_c$ are related to $\overline{\theta}_a$, the assumed constant air temperature by the steady state equation

$$\frac{K}{l}(\overline{\theta}_r - \overline{\theta}_c) = h(\overline{\theta}_c - \overline{\theta}_a)$$

or

$$\frac{h}{K} = \frac{(\overline{\theta}_r - \overline{\theta}_c)}{(\overline{\theta}_c - \overline{\theta}_a)} \times \frac{1}{l}. \tag{3}$$

PROCEDURE

The observed experimental time temperature curves of the roof (θ_r), the ceiling (θ_c) and the indoor ambient air (θ_a) are first analysed into their Fourier components as follows:

$$\begin{aligned} \theta_r = 86{\cdot}07 &+ 23{\cdot}02 \cos(\omega t - 214^\circ\, 9') \\ &+ 6{\cdot}85 \cos(2\omega t - 27^\circ\, 59') \\ &+ 1{\cdot}01 \cos(3\omega t - 115^\circ\, 50') \\ &+ 0{\cdot}50 \cos(4\omega t - 130^\circ\, 22') \\ &+ \ldots\ldots\ldots \end{aligned} \tag{4}$$

$$\begin{aligned} \theta_c = 84{\cdot}33 &+ 14{\cdot}95 \cos(\omega t - 242^\circ\, 35') \\ &+ 3{\cdot}66 \cos(2\omega t - 70^\circ\, 30') \\ &+ 0{\cdot}32 \cos(3\omega t - 163^\circ\, 50') \\ &+ 0{\cdot}25 \cos(4\omega t - 139^\circ\, 31') \\ &+ \ldots\ldots\ldots \end{aligned} \tag{5}$$

$$\begin{aligned} \theta_a = 81{\cdot}45 &+ 6{\cdot}50 \cos(\omega t - 259^\circ\, 54') \\ &+ 1{\cdot}76 \cos(2\omega t - 84^\circ\, 48') \\ &+ 0{\cdot}71 \cos(3\omega t - 56^\circ\, 44') \\ &+ 0{\cdot}34 \cos(4\omega t - 117^\circ\, 17') \\ &+ \ldots\ldots\ldots \end{aligned} \tag{6}$$

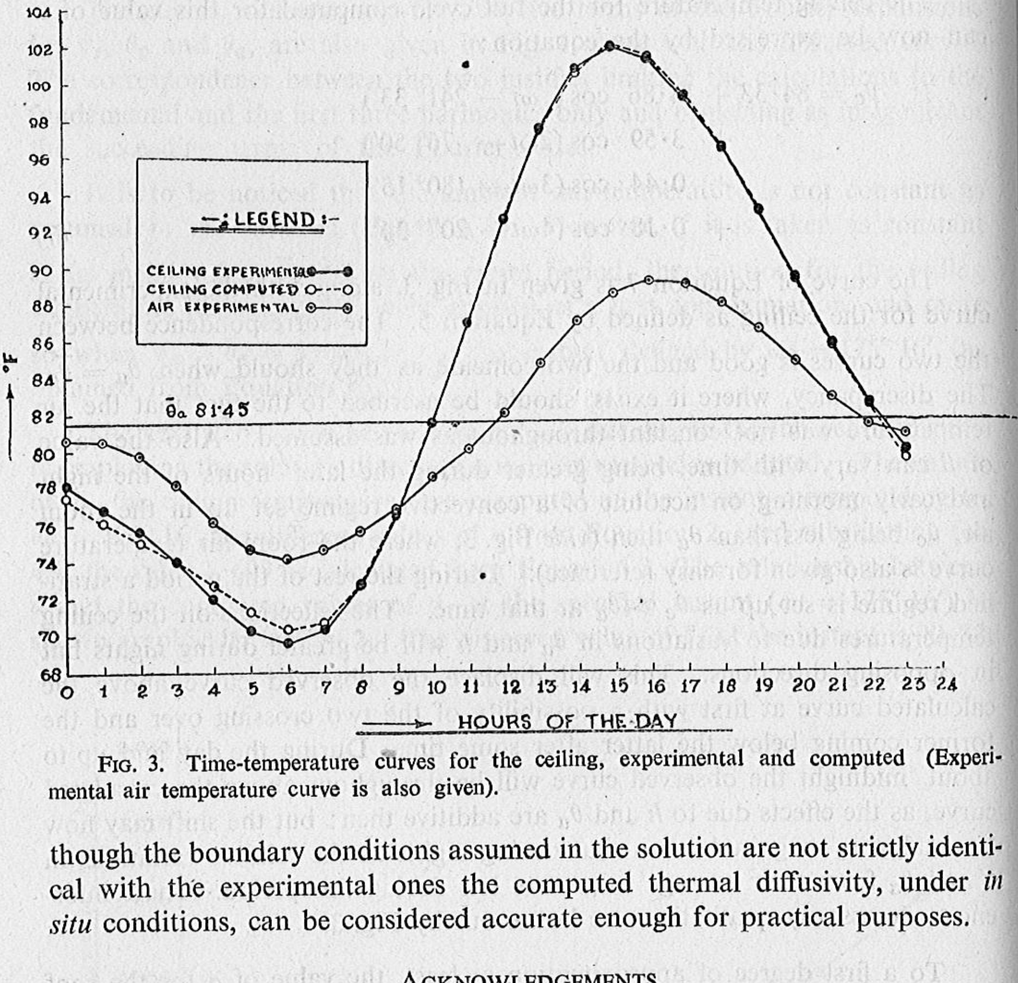

FIG. 3. Time-temperature curves for the ceiling, experimental and computed (Experimental air temperature curve is also given).

though the boundary conditions assumed in the solution are not strictly identical with the experimental ones the computed thermal diffusivity, under *in situ* conditions, can be considered accurate enough for practical purposes.

ACKNOWLEDGEMENTS

Thanks are due to Prof. T. N. Seshadri, Asst. Director, for his sustained interest in this work and his valuable help by way of useful discussions and suggestions, to Shri D. P. Garg for his assistance in the numerical computations and to the Director, Central Building Research Institute, for his kind permission to publish this paper.

REFERENCES

1. Houghton, F. C., Blackshaw, J. L., Pugh, E. M. and McDermott, P. *Trans. Am. Soc. Heat. Vent. Eng.*, 1932, **38**, 260.
2. Raychaudhuri, B. C. .. *Report on Low Cost Housing*, Government of India, New Delhi, January 1957, p. 61.